Overview of Lecture 5 (part of Lecture 4 in Reader book)

- Bond price listings and Yield to Maturity
  - Treasury Bills
  - Treasury Notes and Bonds

- Inflation, Real and Nominal Interest Rates

Bonds: review

- A bond is defined by its **maturity date**, **coupon rate**, and **face (par) value**.
  - The bond makes coupon payments every 6 months until maturity.
  - Each coupon payment = ½ (coupon rate)(face value)
  - The face value is paid out at the maturity date (together with the last coupon payment).

Treasury Bills

- Treasury Bills are issued by the US Treasury, with 3 months, 6 months and 1 year to maturity
  - 3 and 6 month Bills usually auctioned weekly.
  - 1 year Bills usually auctioned monthly.
- Treasury Bills are "zero coupon" bonds.
  - You pay for them today, and receive the face value, usually $10,000, at maturity.
  - Bond prices are quoted relative to a $100 face value
    - » E.g. the quoted price is for 1/100 of a T-Bill

T-Bill Listings
(WSJ, Friday 9/8/00)

TREASURY BILLS

Maturity	Days to Mat.	Bid	Asked	Chg.	Ask Yld.
Sep 14 '00	6	6.28	6.20	+0.02	6.29
Sep 21 '00	13	6.38	6.30	+0.11	6.40
Sep 28 '00	20	5.91	5.83	+0.03	5.93
Oct 05 '00	27	5.96	5.88	−0.01	5.99
Oct 12 '00	34	6.00	5.96	−0.04	6.08

- The **bid price** (at which you can sell) and **ask price** (at which you can buy) are quoted as a **discount** from the face value, **d**.
  - Are these true interest rates?
  - What about the "Ask Yield"?

Treasury Bills: Discount

- Given the quoted discount, we can calculate the price:

$$P = 100 \times \left(1 - \frac{n \times d}{360}\right).$$

- E.g. the bill with maturity 27 days, quoted (ask) discount 5.88%, has price equal to

$$P = 100 \times \left(1 - \frac{n \times d}{360}\right) = 100 \times \left(1 - \frac{27 \times .0588}{360}\right) = \$99.559.$$

- Remember: this is the price per $100 principal.
- So for 1 bill you'd pay 99.559 x 10,000/100 = $9,955.90.

Treasury Bills: True Interest Rate

- The interest rate over the next 27 days is
  - $99.559\,(1+r_{27\text{ day}}) = 100.$
  - I.e. $r_{27\text{ day}} = 100/99.559 - 1 = 0.443\%.$
- This can be converted to an annual interest rate:
  - $1+r_{annual} = (1+r_{27\text{ day}})^{365/27} = 1.0616.$ $r_{annual} = 6.16\%$
- Thus, the true (effective) annual interest rate is slightly higher than the quoted discount rate.

Treasury Bills: Yield

- The "Ask Yield" quoted in the paper is based on the ask price:

$$y = \frac{365}{n} \times \frac{\left(100 - P^{ask}\right)}{P^{ask}} = \frac{d \times 365/360}{1 - nd/360}.$$

- For our bill, y = 365/27 x (100–99.559) / 99.559 = 5.99%.
  - This is larger than the discount, but also smaller than the true interest rate.
  - Note that $(100 - P^{ask})/P^{ask}$ **is** the true n-day interest rate
  - It's then converted to an annual rate by multiplying by 365/n, rather than raising to the power of 365/n

Powerpoint Treasury Bill Calculator

- Enter T-Bill quotes from the Wall Street Journal:

Maturity	Days	Bid	Asked	Chg	Ask Yld
Oct 05 '00	27	5.96	5.88	-0.01	5.99

- Price, $P^{ask} = 100 \times (1 - nd / 360)$ = 99.559
- True r = $(100 - P) / P$ = 0.4429534246025
- Annual r_A = $(1 + r)^{365/n}$ = 6.15695379742831
- Ask yield, $y = (365/n) \times (100 - P) / P$ = 5.98807407333009

Treasury Notes and Bonds

- **Treasury Notes** and **Treasury Bonds** are coupon paying bonds issued by the US government. The only difference is maturity:
  - Notes have more than 1, and up to 10 yrs. to maturity.
  - Bonds have more than 10, and up to 30, yrs. to maturity
- Coupon payments are made every 6 months.

Treasury Bond and Note Listings

(WSJ, Friday 9/8/00)

GOVT. BONDS & NOTES

Rate	Maturity Mo/Yr	Bid	Asked	Chg.	Ask Yld.
4 1/2	Sep 00n	99:26	99:28	....	6.46
6 1/8	Sep 00n	99:30	100:00	....	5.96
4	Oct 00n	99:20	99:22	+ 1	6.10
5 3/4	Oct 00n	99:28	99:30	....	6.06
5 3/4	Nov 00n	99:27	99:29	....	6.15
8 1/2	Nov 00n	100:11	100:13	....	6.11
4 5/8	Nov 00n	99:18	99:20	....	6.22
5 5/8	Nov 00n	99:26	99:28	+ 1	6.09
4 5/8	Dec 00n	99:13	99:15	....	6.32
5 1/2	Dec 00n	99:21	99:23	....	6.36
4 1/2	Jan 01n	99:08	99:10	....	6.26
5 1/4	Jan 01n	99:17	99:19	....	6.27
5 3/8	Feb 01n	99:18	99:20	....	6.24
7 3/4	Feb 01n	100:18	100:20	....	6.24
11 3/4	Feb 01	102:09	102:11	-1	6.17
5	Feb 01n	99:12	99:14	....	6.21
5 5/8	Feb 01n	99:21	99:23	....	6.22
4 7/8	Mar 01n	99:05	99:07	....	6.31
6 3/8	Mar 01n	99:31	100:01	....	6.31

Rate	Maturity Mo/Yr	Bid	Asked	Chg.	Ask Yld.
11 3/4	Nov 09-14	139:14	139:20	-10	6.06
11 1/4	Feb 15	149:24	149:30	-17	6.02
10 5/8	Aug 15	144:23	144:29	-16	6.02
9 7/8	Nov 15	137:27	138:01	-17	6.02
9 1/4	Feb 16	132:05	132:11	-14	6.01
7 1/4	May 16	112:13	112:17	-13	6.00
7 1/2	Nov 16	115:04	115:08	-14	6.01
8 3/4	May 17	128:13	128:19	-14	6.01
8 7/8	Aug 17	129:30	130:04	-16	6.01
9 1/8	May 18	133:13	133:19	-14	6.01
9	Nov 18	132:18	132:24	-14	6.01
8 7/8	Feb 19	131:14	131:20	-14	6.01
8 1/8	Aug 19	123:19	123:25	-13	6.01
8 1/2	Feb 20	128:07	128:13	-14	6.00
8 3/4	May 20	131:08	131:14	-13	6.00
8 3/4	Aug 20	131:16	131:22	-14	6.00
7 7/8	Feb 21	121:27	122:01	-12	5.99
8 1/8	May 21	124:29	125:03	-12	5.99
8 1/8	Aug 21	125:03	125:09	-12	5.99
8	Nov 21	123:27	124:01	-12	5.98

- Prices are quoted in 32nds of a dollar.
  - E.g. 138:01 means \$138 + 1/32 = \$138.03125
- Quoted ("clean") prices are not the actual prices you pay.
  - You have to add "accrued interest"

Accrued Interest

- If t_0 is the date of the last coupon payment, t_1 the date of the next payment, and t is today's date,

$$\text{Accrued interest} = \left(\frac{t - t_0}{t_1 - t_0}\right) \times \text{coupon payment}$$

- For our bond, last coupon was May 15, 2000
  - 116 days ago as of September 8, 2000.
- Next coupon is November 15, 2000
  - 184 days after May 15.
- Accrued interest = (116 / 184) x (9.875 / 2) = $3.11277.
- Price you'd actually pay = $138.03125 + $3.11277.
  = $141.14402.

Yield to Maturity

- The "Ask Yield" column show's the bond's yield to maturity (YTM).
- For annual coupon paying bonds, YTM solves the following equation for y

$$\text{Bond Price} = \frac{C_1}{1+y} + \frac{C_2}{(1+y)^2} + \frac{C_3}{(1+y)^3} + \ldots$$

where the C's are the bond's cash payments.

- For semi-annual coupon paying bonds, it is standard to quote the yield as an APR compounded semiannually:

$$\text{Bond Price} = \frac{C_{6m}}{1+y/2} + \frac{C_{1y}}{(1+y/2)^2} + \frac{C_{18m}}{(1+y/2)^3} + \ldots$$

- Some of you may recognize that the YTM equals the internal rate of return (IRR) for the bond.

Yield to Maturity

- The YTM represents (roughly) the average return to an investor that purchases the bond and holds it until maturity.
- However, the YTM is not a spot or forward interest rate.
  - Spot and forward interest rates relate to investments with just 2 cash flows: at the start date and at the end date.
  - The bond, however, has a sequence of cash flows.
  - The YTM is a sort of weighted average of the spot rates.
  - The weights depend (roughly) on the size of the coupon payments and the number of periods until maturity.
  - What is the YTM of a zero coupon bond?
- Though commonly quoted, it is not a very useful measure
  - Only relevant for one bond.
  - See BM p. 677 for warnings regarding YTM.

Real interest rates, nominal interest rates and inflation

- A **nominal cash flow** is simply the number of dollars you pay out or receive.
  - A $1 nominal cash flow will have different **purchasing power** at different dates due to price level changes.
- A **real cash flow** is adjusted for inflation.
  - A real dollar always has the same purchasing power.
- $1 real equals $1 nominal today.

Converting between Real and Nominal cash flows

- To convert a nominal cash flow to a real cash flow, we need to adjust for the decrease in purchasing power:
  - Real cash flow = Nominal cash flow/(1 + inflation rate)^t
- E.g. BM Example p. 48:
  - If you invest $1,000 today at 10%, your nominal payout in 20 years will be $6727.50 (=1000(1.10)^{20}).
  - If the inflation rate over this period is 6% annually, then the real value will be $2097.67 (=6727.50/1.06^{20}).

Converting between Real and Nominal interest rates

- In this example, what was the real return?
  - $(2097.67/1000)^{1/20} - 1 = 3.77\%$
  - Note that 1.0377 equals 1.1 / 1.06.
- General rule (see reader p. 33 for derivation):
  - $(1+r_{real})(1+i) = 1+r$
    - In our example, (1.0377)(1.06) = 1.10.
  - Equivalently, $r_{real} = (r-i)/(1+i)$.
  - NOT $r_{real} = (r-i)$, though it's approximately correct.

Discounting Real and Nominal Cash Flows

The Rule:

– Always discount real cash flows with the real interest rate.

– Always discount nominal cash flows with the nominal interest rate.

Real and Nominal Discounting: Reader Example P. 34

- Real interest rate: $r_{nominal}$ = 12%; inflation rate = 8%
  - Compute $r_{real} = (r_{nominal} - i)/(1 + i) = .04/1.08 = .037037 = 3.7\%$.
- Cash flows: $C_0 = -100$; $C_1 = 50$; C_2, C_3 growing at inf. rate.
  - Nominal: $C_2 = 50(1+.08) = 54$; $C_3 = 50(1.08)^2 = 58.32$
  - Real: $C_1 = 50/1.08 = 46.3$; $C_2 = C_3 = 46.3$
- What is the NPV?
- 1) Discount nominal cash flows with nominal rate:
  - $NPV = -100 + 50/1.12 + 54/1.12^2 + 58.32/1.12^3 = 29.20$.
- 2) Discount real cash flows with real interest rate:
  - $NPV = -100 + 46.3/1.037 + 46.3/1.037^2 + 46.3/1.037^3 = 29.20$.