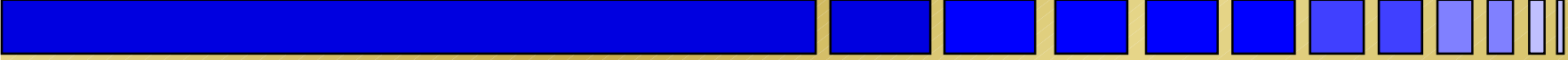
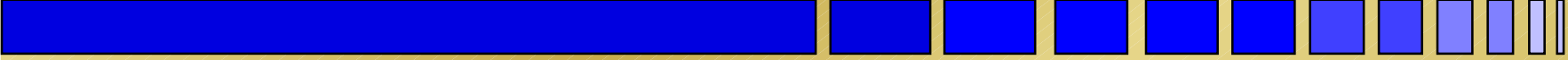


Overview of Lecture 5 (part of Lecture 4 in Reader book)

- 
- Bond price listings and Yield to Maturity
 - Treasury Bills
 - Treasury Notes and Bonds
 - Inflation, Real and Nominal Interest Rates

Bonds: review

- 
- A bond is defined by its **maturity date**, **coupon rate**, and **face (par) value**.
 - The bond makes coupon payments every 6 months until maturity.
 - Each coupon payment = $\frac{1}{2}$ (coupon rate)(face value)
 - The face value is paid out at the maturity date (together with the last coupon payment).

Treasury Bills

- Treasury Bills are issued by the US Treasury, with 3 months, 6 months and 1 year to maturity
 - 3 and 6 month Bills usually auctioned weekly.
 - 1 year Bills usually auctioned monthly.
- Treasury Bills are “zero coupon” bonds.
 - You pay for them today, and receive the face value, usually \$10,000, at maturity.
 - Bond prices are quoted relative to a \$100 face value
 - » E.g. the quoted price is for 1/100 of a T-Bill

T-Bill Listings

(WSJ, Friday 9/8/00)

TREASURY BILLS						
	Days					Ask
Maturity	to	Mat.	Bid	Asked	Chg.	Yld.
Sep 14 '00	6	6.28	6.20	+ 0.02	6.29	
Sep 21 '00	13	6.38	6.30	+ 0.11	6.40	
Sep 28 '00	20	5.91	5.83	+ 0.03	5.93	
Oct 05 '00	27	5.96	5.88	- 0.01	5.99	
Oct 12 '00	34	6.00	5.96	- 0.04	6.08	

- The **bid price** (at which you can sell) and **ask price** (at which you can buy) are quoted as a **discount** from the face value, **d**.
 - Are these true interest rates?
 - What about the “Ask Yield”?

Treasury Bills: Discount

- Given the quoted discount, we can calculate the price:

$$P = 100 \times \left(1 - \frac{n \times d}{360} \right).$$

- E.g. the bill with maturity 27 days, quoted (ask) discount 5.88%, has price equal to

$$P = 100 \times \left(1 - \frac{n \times d}{360} \right) = 100 \times \left(1 - \frac{27 \times .0588}{360} \right) = \$99.559.$$

- Remember: this is the price per \$100 principal.
- So for 1 bill you'd pay $99.559 \times 10,000/100 = \$9,955.90$.

Treasury Bills: True Interest Rate

- The interest rate over the next 27 days is
 - $99.559 (1+r_{27 \text{ day}}) = 100$.
 - I.e. $r_{27 \text{ day}} = 100/99.559 - 1 = 0.443\%$.
- This can be converted to an annual interest rate:
 - $1+r_{\text{annual}} = (1+r_{27 \text{ day}})^{365/27} = 1.0616$. $r_{\text{annual}} = 6.16\%$
- Thus, the true (effective) annual interest rate is slightly higher than the quoted discount rate.

Treasury Bills: Yield

- The “Ask Yield” quoted in the paper is based on the ask price:

$$y = \frac{365}{n} \times \frac{(100 - P^{\text{ask}})}{P^{\text{ask}}} = \frac{d \times 365 / 360}{1 - nd / 360}.$$

- For our bill, $y = 365/27 \times (100 - 99.559) / 99.559 = 5.99\%$.
 - This is larger than the discount, but also smaller than the true interest rate.
 - Note that $(100 - P^{\text{ask}}) / P^{\text{ask}}$ is the true n-day interest rate
 - It’s then converted to an annual rate by multiplying by $365/n$, rather than raising to the power of $365/n$

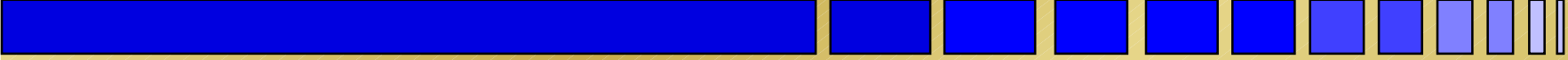
Powerpoint Treasury Bill Calculator

- Enter T-Bill quotes from the Wall Street Journal:

<u>Maturity</u>	<u>Days</u>	<u>Bid</u>	<u>Asked</u>	<u>Chg</u>	<u>Ask Yld</u>
Oct 05 '00	27	5.96	5.88	-0.01	5.99

- Price, $P^{\text{ask}} = 100 \times (1 - nd / 360) = 99.559$
- True $r = (100 - P) / P = 0.4429534246025$
- Annual $r_A = (1 + r)^{365/n} = 6.15695379742831$
- Ask yield, $y = (365/n) \times (100 - P) / P = 5.98807407333009$

Treasury Notes and Bonds

- 
- **Treasury Notes** and **Treasury Bonds** are coupon paying bonds issued by the US government. The only difference is maturity:
 - Notes have more than 1, and up to 10 yrs. to maturity.
 - Bonds have more than 10, and up to 30, yrs. to maturity
 - Coupon payments are made every 6 months.

Treasury Bond and Note Listings

(WSJ, Friday 9/8/00)

GOVT. BONDS & NOTES						Maturity			Ask		
Rate	Maturity Mo/Yr	Bid	Asked	Chg.	Yld.	Rate	Mo/Yr	Bid	Asked	Chg.	Yld.
4 1/2	Sep 00n	99:26	99:28	6.46	11 3/4	Nov 09-14	139:14	139:20	-10	6.06
6 1/8	Sep 00n	99:30	100:00	5.96	11 1/4	Feb 15	149:24	149:30	-17	6.02
4	Oct 00n	99:20	99:22	+ 1	6.10	10 5/8	Aug 15	144:23	144:29	-16	6.02
5 3/4	Oct 00n	99:28	99:30	6.06	9 7/8	Nov 15	137:27	138:01	-17	6.02
5 3/4	Nov 00n	99:27	99:29	6.15	9 1/4	Feb 16	132:05	132:11	-14	6.01
8 1/2	Nov 00n	100:11	100:13	6.11	7 1/4	May 16	112:13	112:17	-13	6.00
4 5/8	Nov 00n	99:18	99:20	6.22	7 1/2	Nov 16	115:04	115:08	-14	6.01
5 5/8	Nov 00n	99:26	99:28	+ 1	6.09	8 3/4	May 17	128:13	128:19	-14	6.01
4 5/8	Dec 00n	99:13	99:15	6.32	8 7/8	Aug 17	129:30	130:04	-16	6.01
5 1/2	Dec 00n	99:21	99:23	6.36	9 1/8	May 18	133:13	133:19	-14	6.01
4 1/2	Jan 01n	99:08	99:10	6.26	9	Nov 18	132:18	132:24	-14	6.01
5 1/4	Jan 01n	99:17	99:19	6.27	8 7/8	Feb 19	131:14	131:20	-14	6.01
5 3/8	Feb 01n	99:18	99:20	6.24	8 1/8	Aug 19	123:19	123:25	-13	6.01
7 3/4	Feb 01n	100:18	100:20	6.24	8 1/2	Feb 20	128:07	128:13	-14	6.00
11 3/4	Feb 01	102:09	102:11	-1	6.17	8 3/4	May 20	131:08	131:14	-13	6.00
5	Feb 01n	99:12	99:14	6.21	8 3/4	Aug 20	131:16	131:22	-14	6.00
5 5/8	Feb 01n	99:21	99:23	6.22	7 7/8	Feb 21	121:27	122:01	-12	5.99
4 7/8	Mar 01n	99:05	99:07	6.31	8 1/8	May 21	124:29	125:03	-12	5.99
6 3/8	Mar 01n	99:31	100:01	6.31	8 1/8	Aug 21	125:03	125:09	-12	5.99
						8	Nov 21	123:27	124:01	-12	5.98

- Prices are quoted in 32nds of a dollar.
 - E.g. 138:01 means $\$138 + 1/32 = \138.03125
- Quoted (“clean”) prices are not the actual prices you pay.
 - You have to add “accrued interest”

Accrued Interest

- If t_0 is the date of the last coupon payment, t_1 the date of the next payment, and t is today's date,

$$\text{Accrued interest} = \left(\frac{t - t_0}{t_1 - t_0} \right) \times \text{coupon payment}$$

- For our bond, last coupon was May 15, 2000
 - 116 days ago as of September 8, 2000.
- Next coupon is November 15, 2000
 - 184 days after May 15.
- Accrued interest = $(116 / 184) \times (9.875 / 2) = \3.11277 .
- Price you'd actually pay = $\$138.03125 + \3.11277 .
= $\$141.14402$.

Yield to Maturity

- The “Ask Yield” column show’s the bond’s **yield to maturity** (YTM).
- For annual coupon paying bonds, YTM solves the following equation for y

$$\text{Bond Price} = \frac{C_1}{1+y} + \frac{C_2}{(1+y)^2} + \frac{C_3}{(1+y)^3} + \dots$$

where the C 's are the bond's cash payments.

- For semi-annual coupon paying bonds, it is standard to quote the yield as an APR compounded semiannually:

$$\text{Bond Price} = \frac{C_{6m}}{1+y/2} + \frac{C_{1y}}{(1+y/2)^2} + \frac{C_{18m}}{(1+y/2)^3} + \dots$$

- Some of you may recognize that the YTM equals the **internal rate of return** (IRR) for the bond.

Yield to Maturity

- The YTM represents (roughly) the average return to an investor that purchases the bond and holds it until maturity.
- However, the YTM is **not** a spot or forward interest rate.
 - Spot and forward interest rates relate to investments with just 2 cash flows: at the start date and at the end date.
 - The bond, however, has a sequence of cash flows.
 - The YTM is a sort of weighted average of the spot rates.
 - The weights depend (roughly) on the size of the coupon payments and the number of periods until maturity.
 - What is the YTM of a zero coupon bond?
- Though commonly quoted, it is not a very useful measure
 - Only relevant for one bond.
 - See BM p. 677 for warnings regarding YTM.

Real interest rates, nominal interest rates and inflation

- A **nominal cash flow** is simply the number of dollars you pay out or receive.
 - A \$1 nominal cash flow will have different **purchasing power** at different dates due to price level changes.
- A **real cash flow** is adjusted for inflation.
 - A real dollar always has the same purchasing power.
- \$1 real equals \$1 nominal today.

Converting between Real and Nominal cash flows

- To convert a nominal cash flow to a real cash flow, we need to adjust for the decrease in purchasing power:
 - Real cash flow = Nominal cash flow / (1 + inflation rate)^t
- E.g. BM Example p. 48:
 - If you invest \$1,000 today at 10%, your nominal payout in 20 years will be \$6727.50 (=1000(1.10)²⁰).
 - If the inflation rate over this period is 6% annually, then the real value will be \$2097.67 (=6727.50/1.06²⁰).

Converting between Real and Nominal interest rates

- In this example, what was the real return?
 - $(2097.67/1000)^{1/20} - 1 = 3.77\%$
 - Note that 1.0377 equals $1.1 / 1.06$.
- General rule (see reader p. 33 for derivation):
 - $(1+r_{\text{real}})(1+i) = 1+r$
 - » In our example, $(1.0377)(1.06) = 1.10$.
 - Equivalently, $r_{\text{real}} = (r-i)/(1+i)$.
 - NOT $r_{\text{real}} = (r-i)$, though it's approximately correct.

Discounting Real and Nominal Cash Flows

The Rule:

- Always discount **real** cash flows with the **real** interest rate.
- Always discount **nominal** cash flows with the **nominal** interest rate.

Real and Nominal Discounting: Reader Example P. 34

- Real interest rate: $r_{\text{nominal}} = 12\%$; inflation rate = 8%
 - Compute $r_{\text{real}} = (r_{\text{nominal}} - i)/(1 + i) = .04/1.08 = .037037 = 3.7\%$.
- Cash flows: $C_0 = -100$; $C_1 = 50$; C_2, C_3 growing at inf. rate.
 - Nominal: $C_2 = 50(1+.08) = 54$; $C_3 = 50(1.08)^2 = 58.32$
 - Real: $C_1 = 50/1.08 = 46.3$; $C_2 = C_3 = 46.3$
- What is the NPV?
- 1) Discount nominal cash flows with nominal rate:
 - $\text{NPV} = -100 + 50/1.12 + 54/1.12^2 + 58.32/1.12^3 = 29.20$.
- 2) Discount real cash flows with real interest rate:
 - $\text{NPV} = -100 + 46.3/1.037 + 46.3/1.037^2 + 46.3/1.037^3 = 29.20$.