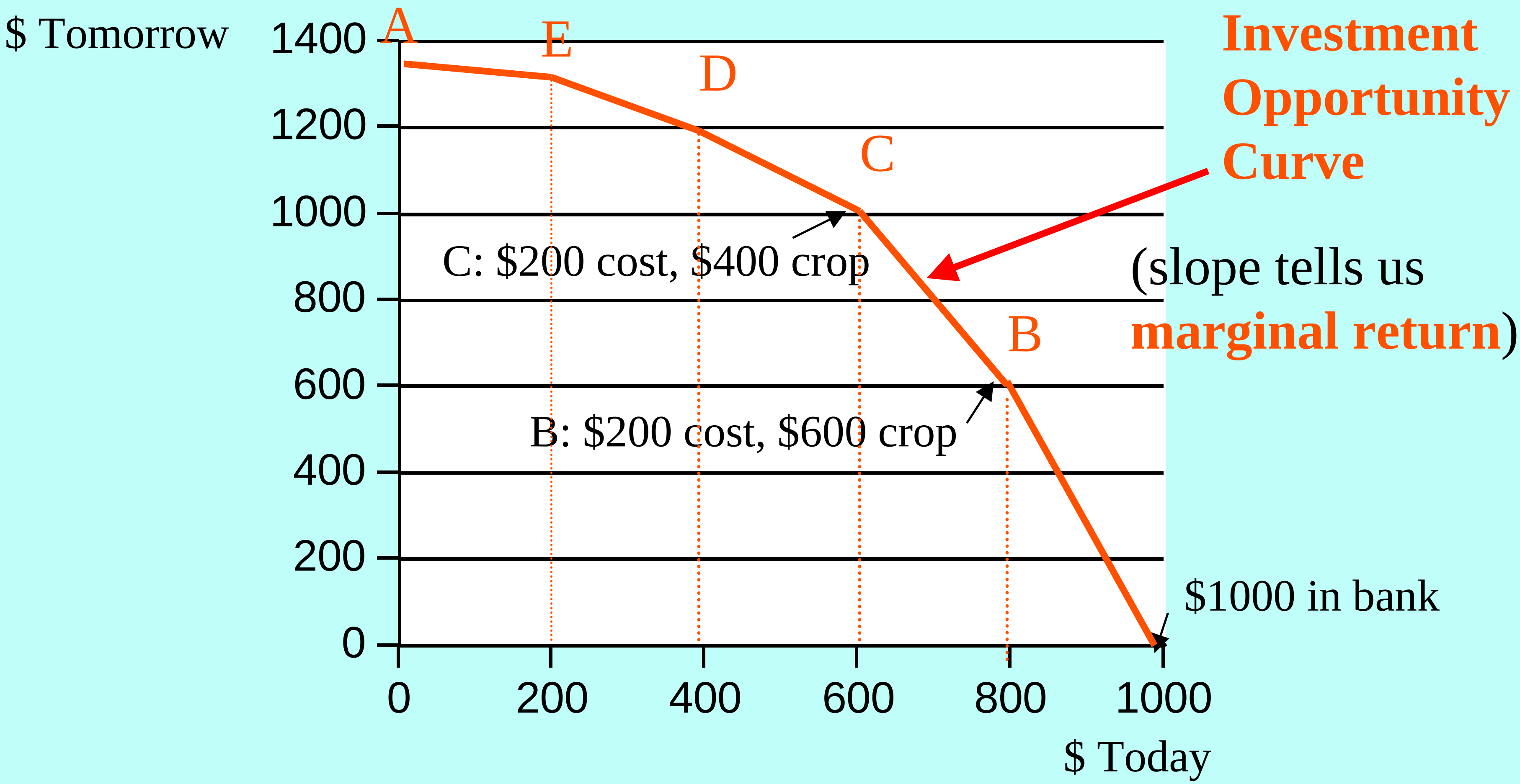
Overview of Next Topic

- Choosing among multiple projects
  - Investment opportunity curve
  - Optimal investment decisions
  - Maximizing PV as criterion for investment choice by firm
- Calculating Present Value
- Interest rates
  - Definition
  - Compounding
    - going from short to long periods
    - going from long to short periods

Choosing Among Many Projects

- You set up a firm whose assets are $1,000 in the bank and a corn farm.
  - Each column of corn costs $200 to seed.
  - There are 5 columns.
- The payoff for each column is:
  - Row A: 50, B: 600, C: 400, D: 200, E: 100
- Plot the tradeoff between dividends today and dividends tomorrow.

Planting Columns of Corn



Firm's Optimal Investment

- Managers should thus
  - Take on every project with a return > r (positive NPV)
  - Reject every project with a return < r (negative NPV)
  - I.e. they should use the **NPV rule**.
- This maximizes the PV of the firm.
  - All shareholders agree on this investment policy.
  - For alternative views of managers' objectives, see Micro/OB
- Note: the firm's PV exceeds the cash it starts with.
  - Every time it invests in a + NPV project, the firm's PV increases by the project's NPV.

What discount rate to use?

- So far, we have assumed everything is riskless.
- Our discount rate, r has been the riskless interest rate.
- More generally, r represents the **opportunity cost of capital.**
  - The expected return on other, "equivalently risky" investments.
  - We'll see how to calculate this later in the course

- <u>Idea</u>: A project only makes us better off if its return is higher than the return we could get on alternative investments elsewhere.
  - NPV quantifies how much better off.

PV with multiple cash flows

- The present value (PV) of a cash flow C_1 one year from today is

$$PV = C_1/(1+r) = DF_1 \times C_1$$

where $DF_1 = 1 / (1+r)$, the **discount factor** for period 1 (the present value of \$1).

- For cash flow in year t, $PV = C_t / (1+r)^t = DF_t \times C_t$.
- For cash flows in several periods, C_0, C_1, C_2, …,

$$PV = C_0 + \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \ldots$$

Example

- $C_0 = -11$
- $C_1 = 6$
- $C_2 = 7$
- $r = .09$

$$
\begin{aligned}
\text{(N)PV} &= -11 + \frac{6}{1.09} + \frac{7}{(1.09)^2} \\
&= -11 + 5.50 + 5.89 \\
&= 0.40.
\end{aligned}
$$

Example - Using a Financial Calculator (e.g. HP-12C)

- Step 1: Clear all registers ([f][REG] on HP-12C)
- Step 2: Enter data:

{11}[CHS][g][CF_0]	(The period 0 cash flow)
{6}[g][CF_j]	(The period 1 cash flow)
{7}[g][CF_j]	(The period 2 cash flow)
{9}[i]	(The interest rate)
[f][NPV]	(Asks for the answer)

Example: A Lottery

- You know that the probability of winning the lottery is 1 in 13 million.
- Walking by the news stand you see a big sign, "JACKPOT 20 MILLION!![1]"
- Should you invest $1 for a ticket?

[1]Paid in 20 annual equal installments.
Payments are tax-free.

Lottery Decision

- First step: calculate the present value of the payoff.
  - A quick examination of the *Wall Street Journal* shows that the appropriate interest rate is 8%.

$$PV = \frac{\$1M}{1.08^1} + \frac{\$1M}{1.08^2} + \frac{\$1M}{1.08^3} + \ldots + \frac{\$1M}{1.08^{20}} = \$9.8M$$

- Odds are 1 in 13 M, so PV of expected payoff is $9.8M/13 M =$.75.
- NPV = 0.75 – 1.00 = -$0.25
- Maybe next time…

Basics of Interest Rates

- **Definition**: Let r_n represents an **n period interest rate**. If you invest C dollars for n periods, you will end up with $C(1+r_n)$ dollars.
- For the purposes of this course, only numbers that meet the above definition are interest rates.

Compounding: How to go from short periods to long periods

- Example: The 1 month interest rate is 1%. What is the 1 year rate?
- There are 12 months in 1 year.

$(1.01)^{12}$	=	$(1+r_{yearly})$
Interest received after 12 months in the bank.		Interest received after 1 year in the bank.

- $r_{yearly} = (1.01)^{12} - 1 = .1268 = 12.68\%$.

Compounding: How to go from long periods to short periods

- Example: The annual interest rate is 14%. What is the daily rate?
- In 1 year there are 365 days.

1.14	=	$(1+r_{daily})^{365}$
Interest received		Interest received
after 1 year		after 365 days
in the bank.		in the bank.

- $r_{daily} = 1.14^{1/365} - 1 = .000359 = 0.0359\%$