



# Lecture 19

## Capital Structure with Taxes: APV and WACC

- Readings:
  - BM, Chapter 19
  - Reader, Lecture 19

# Where are we so far?

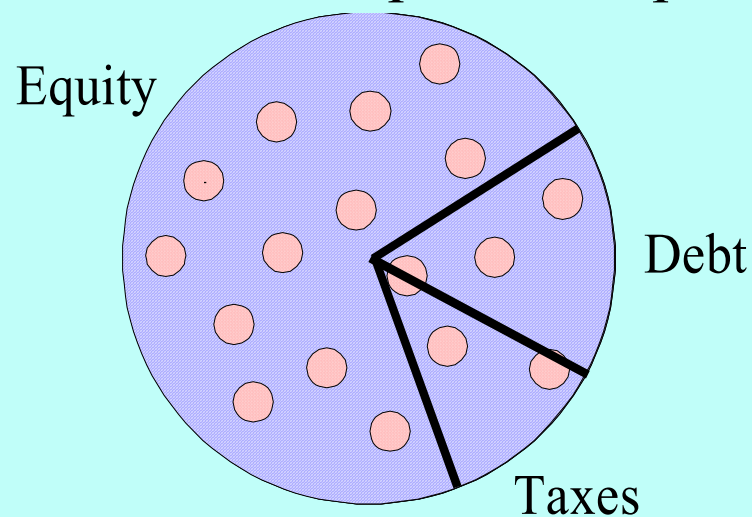
- MM without taxes: Financial policy is irrelevant.
  - Debt/Equity: indifferent (size of pizza doesn't change)
  - Dividends today vs. later: indifferent.
- More leverage increases risk (beta) of equity
  - Total firm risk (i.e. asset beta) remains the same.
  - As we increase debt/reduce equity, total risk is shared among fewer equity holders.
- Converting between equity and asset betas:

$$\beta_A = \beta_E \left( \frac{E_L}{D_L + E_L} \right) + \beta_D \left( \frac{D_L}{D_L + E_L} \right), \quad \beta_E = \beta_A \left( 1 + \frac{D_L}{E_L} \right) - \beta_D \left( \frac{D_L}{E_L} \right).$$

# Capital Structure with Taxes

- With taxes, cash flows now split into 3 pieces:

- Equity
- Debt
- Taxes.



- By reducing tax payments, we increase the combined size of the equity/debt slices.
- Interest on debt is tax deductible; dividends are not.

# Example of Tax Benefit of Interest Payments on Debt

- Net income before tax = \$100; tax rate = 30%.
- Without debt:
  - Taxes paid are **\$30** ( $= 100 \times .30$ ).
  - After tax cash flow = dividends =  $100 - 30 =$  **\$70**
- With debt (interest payments of \$20 per year):
  - Operating Income 100
  - Interest - 20
  - Income before tax 80
  - Tax ( $.30 \times 80$ ) **\$24**
  - After tax cash flow **\$76** ( $= \$20$  interest + \$56 dividends)
- **Debt tax shield** = gain in after tax income =  $76 - 70 =$  \$6.
- This equals (interest paid)  $\times$  (tax rate) =  $20 \times .30 =$  \$6.

# Valuation with Taxes

- There are two methods for valuing a project (i.e. calculating  $V_L = E_L + D_L$  including the value of these tax shields).
- Method 1: **Adjusted Present Value (APV)**
  - Forecast the project’s “all-equity” cash flows (free cash flow + after tax-interest).
  - Discount using  $r_A$  to get the base PV (if it were all equity financed).
  - Finally, add the PV of all current and future tax shields.
- Method 2: **Weighted Average Cost of Capital (WACC)**
  - Forecast the all-equity cash flows as above
  - Discount using WACC instead of  $r_A$  (see handout for explanation).
- Which method you use depends on your assumptions about future debt levels.

# Assumptions about Future Debt

Debt = Constant proportion

Debt = Constant \$ amount

Easy

Hard

Easy

WACC

APV

# Converting between Equity betas and Asset betas (with Taxes)

- For Capstone, you may wish to compare your firm's beta with the beta of other firms.
  - If all firms are unlevered, the equity betas are directly comparable.
  - If firms are levered by differing amounts, then you cannot directly compare their equity betas—it is comparing apples and oranges.
  - The solution is to convert each firm's equity beta to its asset beta (which is equivalent to the beta of an unlevered firm). You can then compare the asset betas directly.
  - You can also average the asset betas, if you feel this provides a better estimate than the asset beta directly estimated for your firm.
- Using APV, we need  $\beta_A$  and  $r_A$  to value the “all-equity” firm.
- The formulae for converting between equity and asset betas are provided on the next page.

# Converting between Equity and Asset Betas

- Without taxes, the conversion formula is easy:

$$\beta_E = \beta_A \left( 1 + \frac{D_L}{E_L} \right) - \beta_D \left( \frac{D_L}{E_L} \right).$$

- With taxes, the formula to use depends on assumptions about future debt (see handout for details):

- Constant amount: 
$$\beta_E = \beta_A \left[ 1 + \frac{D_L(1-T_C)}{E_L} \right] - \beta_D \left[ \frac{D_L(1-T_C)}{E_L} \right].$$

- Constant proportion: 
$$\beta_E = \beta_A \left[ 1 + \frac{D_L}{E_L} \left( 1 - \frac{T_C r_D}{1+r_D} \right) \right] - \beta_D \frac{D_L}{E_L} \left( 1 - \frac{T_C r_D}{1+r_D} \right).$$

# Valuation using APV

## ■ Adjusted Present Value (APV):

- APV = Base PV (with all equity financing) + PV of all tax shields.
- APV is easiest to use when we assume a **constant amount of debt forever**, but can be used in other cases with enough effort.
  - WACC requires a **constant proportion of debt forever**. It *cannot* be used otherwise.

# Example: APV Valuation of Inertia, Inc.: Step 1

## ■ Assumptions:

- Cash flow before interest/tax in year 1 = \$1,200.
- Growth rate = 6%/year forever.
- $\beta_A = 1.5$ ,  $r_f = 5\%$ , and  $r_m - r_f = 10\%$ .
- So  $r_A = 20\%$  ( $= 5\% + (1.5)(10\%)$ ).
- The firm pays tax at 30%.
- Inertia has \$1,600 of riskless debt; constant forever.

## ■ All-equity value:

- After tax CF =  $1,200 \times (1.0 - .30) = \$840$ , growing 6% annually.
- $V_U = E_U = 840 / (.20 - .06) = \mathbf{\$6,000}$

# APV Step 2: Calculate PV(tax shields)

- $D_L = \$1,600$ .
- Tax shield each period =  $T_C r_D D_L$   
 $= 0.3 \times 0.05 \times 1600$   
 $= \$24$
- PV (tax shields) =  $24 / r_D$   
 $= 24 / .05$   
 $= \$480 (= T_C D_L)$
- **Note:** When there is a constant amount of debt, PV of debt tax shield equals  $T_C D_L$ .

# APV Step 3: Add them together

- So  $V_L = V_U + \text{PV}(\text{tax shields})$   
 $= 6,000 + 480$   
 $= \mathbf{\$6,480}$

- I.e.  $E_L = V_L - D_L$   
 $= 6,480 - 1,600$   
 $= \$4,880.$

# Weighted Average Cost of Capital (WACC)

- This is by far the most common method.
- **It only works if two important assumptions hold:**
  - The firm maintains a constant debt / (debt + equity) ratio over its lifetime.
  - The project has the same risk as the rest of the firm.
- The WACC is defined by:

$$WACC = \frac{D}{D+E} (1 - T_c) r_D + \frac{E}{D+E} r_E, \quad \text{or}$$

$$WACC = r_A - \left( \frac{D}{D+E} \right) T_c r_D \left( \frac{1+r_A}{1+r_D} \right)$$

where  $T_c$  is the firm's tax rate,  $r_D$  the firm's borrowing rate.

- These two expressions are equivalent.

# Example: WACC Valuation of Dynamic Cities: Step 1

## ■ Assume:

- Cash flow before interest/tax in year 1 = \$100.
- Growth rate = 10%/year forever.
- $\beta_A = 2$ ,  $r_f = 10\%$ , and  $r_m - r_f = 5\%$ ; so  $r_A = 20\%$ .
- The firm pays tax at 30%.
- Firm will keep debt/(debt + equity) ratio at 15% forever
  - » Debt is riskless.

## ■ Results

- Without debt, after-tax cash flow is  $100 \times (1 - .3) = \$70$ .
- This is expected to grow at 10% per year.

# WACC step 2: Calculate WACC

- The following formula allows us to convert between  $r_A$  and WACC:

$$\begin{aligned} \text{WACC} &= r_A - \left( \frac{D}{D+E} \right) T_C r_D \left( \frac{1+r_A}{1+r_D} \right), \\ &= .2 - \left( .15 \times .3 \times .1 \times \frac{1.2}{1.1} \right), \\ &= 19.51\%. \end{aligned}$$

- Note that  $\text{WACC} < r_A$ .
- If  $D = 0$  or  $T_C = 0$ ,  $\text{WACC} = r_A$ .

# WACC step 3: Discount all-equity cash flows using WACC

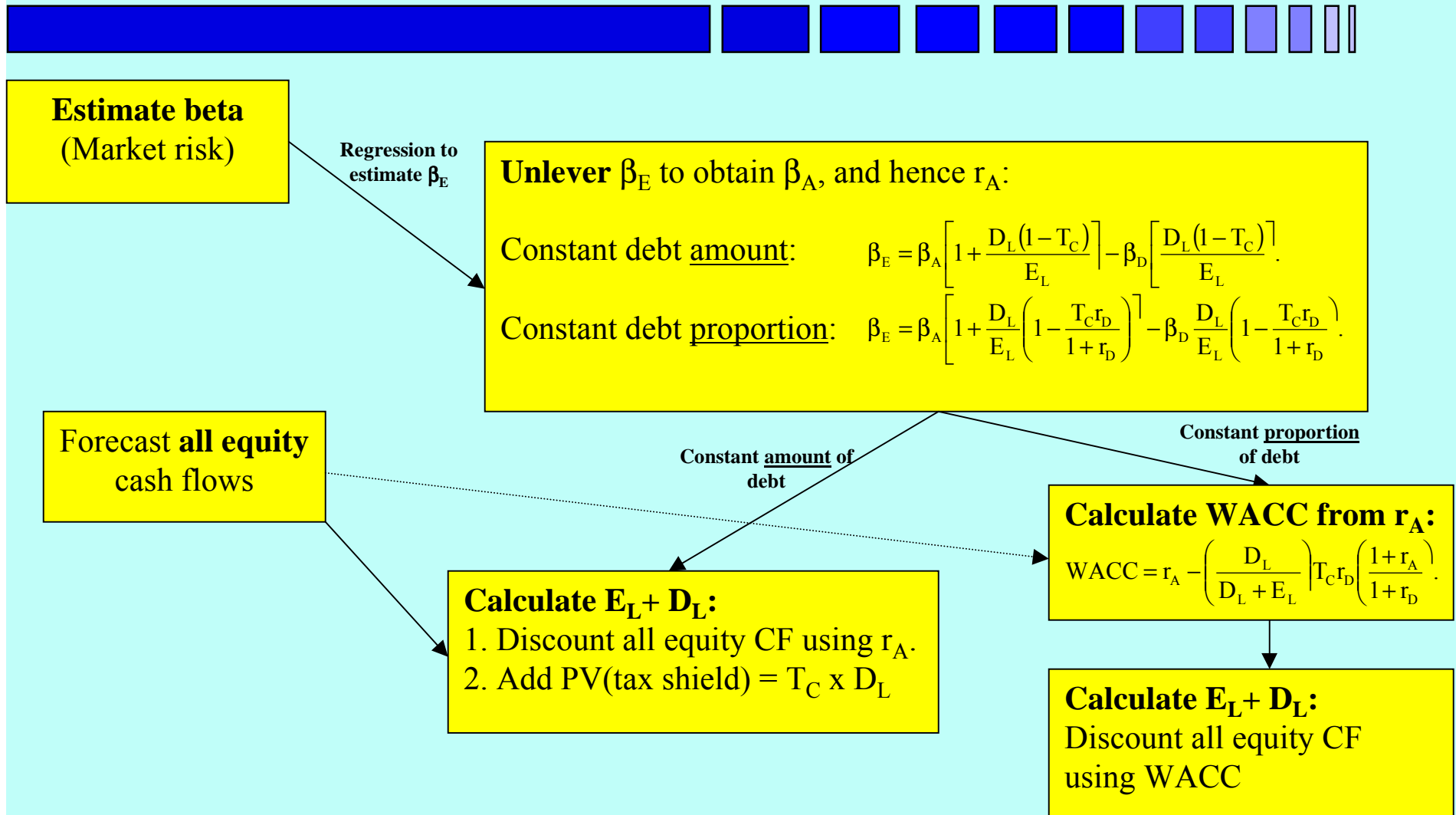
- So  $V_L = E_L + D_L$   
 $= 70 / (.1951 - .1) = \$736.14.$

- Debt is 15% of this total, so

$$D_L = 0.15 \times \$736.14 = \$110.42,$$

$$E_L = 0.85 \times \$736.14 = \$625.72.$$

# Valuation on one slide...



# WACC vs. APV

- We have used:
  - WACC when debt proportion is constant
  - APV when debt amount is constant,
- WACC can only be used if proportion is constant.
- APV can always be used.
  - It may be hard to do the calculations.
- Let's do an example with constant debt proportions, using APV, instead of WACC.
  - Also allows us to see effect on value of constant proportion vs. constant amount of debt.


# Constant amount vs. constant proportion of debt

- For Stagnant Cities Inc.,  $\beta_A = 2$ .
- Also,  $r_f = 10\%$ ,  $r_m - r_f = 5\%$ , so  $r_A = 20\%$ .
- Expected cash flow before interest/tax = \$100.
  - No growth expected.
- Firm pays taxes at a rate of 30%, and currently has \$100 of riskless debt outstanding.
- Let's value the firm using APV:
  - Assuming it will retain the same dollar amount of debt
  - Assuming it will retain the same proportion of debt


# Valuing Stagnant Cities with constant amount of debt

- Assume firm will always have \$100 of debt.
- All equity cash flow =  $\$100 \times (1 - .3) = \$70$
- So  $V_U = E_U = 70 / .2 = \$350$
- PV (tax shields) =  $T_C D_L = .3 \times 100 = \$30$
- So  $E_L = V_L - D_L = E_U + PV(\text{tax shields}) - D_L$   
 $= 350 + 30 - 100$   
 $= \$280$

# Valuing Stagnant Cities with constant proportion of debt

- 
- Now assume the firm will always keep same proportion of debt.
  - In this case (see reader), we can calculate the PV of the tax shields by discounting each by
    - $r_D$  for one period (each is known one period in advance)
    - $r_A$  for remaining periods (amount of debt varies with firm)

# Valuing Stagnant Cities with constant proportion of debt

- 
- $PV(\text{tax shields}) = 3/1.1 + 3/(1.1 \times 1.2) + 3/(1.1 \times 1.2^2) + \dots$   
 $= [3 / (.2)] \times (1.2 / 1.1)$   
 $= \$16.3636$
  - So  $E_L = E_U + PV(\text{tax shields}) - D_L$   
 $= 350 + 16.36 - 100$   
 $= \$266.36$
  - [Compare with previous answer, \$280].


# Valuing Stagnant Cities with constant proportion of debt

- Note that WACC gives the same result:

$$\begin{aligned} \text{WACC} &= r_A - \left( \frac{D}{D+E} \right) T_C r_D \left( \frac{1+r_A}{1+r_D} \right), \\ &= .2 - \left( \frac{100}{100+266.36} \times .3 \times .1 \times \frac{1.2}{1.1} \right), \\ &= 19.1067\%. \end{aligned}$$

- So  $E_L + D_L = 70 / .191067 = \$366.36$ .
- $E_L = \$366.36 - 100 = \$266.36$ .

# Constant amount vs. constant proportion of debt

- 
- In both cases, current amount of debt = \$100.
  - In both cases, expected debt in future = \$100.
  - Yet the equity values are not the same.
    - $E_L$  [equivalently,  $PV(\text{tax shields})$ ] is higher under the constant **amount** of debt assumption.
  - Why?
    - In constant proportion case, amount of debt varies with the size of the firm.
    - Aggregate tax shields are thus risky.

# Using WACC given an initial amount of debt

- In the last example, we knew  $E_L$  before we calculated WACC.
  - Typically, this is how we do it – use market value for  $E_L$
- What if we did not know  $E_L$ , but were given the initial amount, rather than proportion, of debt?
- Problem:
  - We need  $D/(D+E)$  to calculate WACC.
  - We need WACC to calculate  $E$ , and hence  $D/(D+E)$ .

# Summary



**Estimate beta**  
(Market risk)

Regression to estimate  $\beta_E$

**Unlever  $\beta_E$  to obtain  $\beta_A$ , and hence  $r_A$ :**

Constant debt amount: 
$$\beta_E = \beta_A \left[ 1 + \frac{D_L(1-T_C)}{E_L} \right] - \beta_D \left[ \frac{D_L(1-T_C)}{E_L} \right].$$

Constant debt proportion: 
$$\beta_E = \beta_A \left[ 1 + \frac{D_L}{E_L} \left( 1 - \frac{T_C r_D}{1+r_D} \right) \right] - \beta_D \frac{D_L}{E_L} \left( 1 - \frac{T_C r_D}{1+r_D} \right).$$

**Forecast all equity**  
cash flows

Constant amount of debt

Constant proportion of debt

**Calculate  $E_L + D_L$ :**  
1. Discount all equity CF using  $r_A$ .  
2. Add PV(tax shield) =  $T_C \times D_L$

**Calculate WACC from  $r_A$ :**  
$$WACC = r_A - \left( \frac{D_L}{D_L + E_L} \right) T_C r_D \left( \frac{1+r_A}{1+r_D} \right).$$

**Calculate  $E_L + D_L$ :**  
Discount all equity CF using WACC