Lecture 16: Capital Budgeting, Beta, and Cash Flows

- Reading:
  - Brealey and Myers, Chapter 9
  - Lecture Reader, Chapter 15
- Topics:
  - Final topics on basic CAPM
  - Debt, Equity, and Asset Betas
  - Leveraged Betas
  - Operating Leverage

Calculating the beta of a portfolio

- Consider a portfolio of n securities, with weights $w_1, ..., w_n$.
- The beta of the portfolio, β_p, is given by
  $$\beta_p = w_1\beta_1 + w_2\beta_2 + ... + w_n\beta_n.$$
- The expected return on the portfolio is
  $$r_p = w_1 r_1 + w_2 r_2 + ... + w_n r_n$$
  $$= r_f + \beta_p(r_m - r_f).$$

Example
($100 Total Value Portfolio)

Asset	β	w	$
1	0	-.25	-$25
2	.5	1.5	$150
3	1	-.75	-$75
4	1.5	.5	$50

The portfolio β is:

$\beta_p = -.25(0) + 1.5(.5) - .75(1) + .5(1.5) = .75$

Beta and the Market Portfolio

- The market portfolio's beta equals exactly 1 since:

$$\beta_m = \frac{Cov(\tilde{r}_m, \tilde{r}_m)}{Var(\tilde{r}_m)} = \frac{Var(\tilde{r}_m)}{Var(\tilde{r}_m)} = 1$$

- Therefore the weighted sum of all the betas in the economy equals one.

The CAPM in practice (BM 199-211)

- Graham and Harvey (1999) sent a survey to CFOs of all Fortune 500 companies, plus the 4,440 members of the Financial Executives Institute.
- Main findings:
  - 74.9% of respondents (almost) always use **NPV!!**
    - » Compares with 9.8% found by Gitman/Forrester (1977).
  - 73.5% set discount rate using **CAPM!!**
  - 58.8% would use a single company-wide discount rate for all projects, regardless of type..
- See BM pp. 199-211 for more discussion

Betas and Cash Flows

- Any cash flow can be an asset with its own β.
- For example, a company with many divisions may have a different β for each division's revenues.
  - a division with β = 1.5 will require a higher discount rate than a division with β = .75.
- Even individual machine cash flows may have β:
  - Further, the machine's expense cash flows and its revenue cash flows may have different β.
- Now, what effect does Debt have on β?

Debt, Equity and Asset Betas

- The profits generated by a firm's assets are distributed to its debt and equity holders.
- Therefore, one can think of a firm's assets as equivalent to a portfolio of debt and equity.
  - A = dollar value of the firm's assets.
  - D = dollar value of the firm's debt.
  - E = dollar value of the firm's equity.
- By accounting definition: A = D + E.

The Asset Beta

- Since A = D+E, we can write the asset beta as a function of the debt and equity betas:

$$\beta_A = \beta_D \frac{D}{D+E} + \beta_E \frac{E}{D+E} = (d)(\beta_D) + (e)(\beta_E),$$

$$\text{where } d = \frac{D}{D+E};\ e = \frac{E}{D+E}.$$

- We can also write $\beta_E = (\beta_A - d\beta_d)/(e)$
- If firm's debt is risk free, equity beta has form: $\beta_E = \beta_A/e = \beta_A A/E = \beta_A A/(A\text{-}D)$.

The Effect of Risk-Free Debt on Beta

- A firm's asset beta depends on the assets alone; it does not change as the amount of debt changes.
- But a firm's equity beta does depend on its risk-free debt: $\beta_E = \beta_A/e = \beta_A A/E = \beta_A A/(A-D)$.
  - The linkage between the firm's equity beta and its debt-equity mix is often overlooked.
  - In fact, we see that β_E rises in tandem with debt D.
  - What does this tell you about the effect of more debt on the firm's riskiness?

Is Debt Less Costly then Equity? (No!)

- People often think debt is less costly than equity, since bond interest rates < expected stock returns.
  - This misses point that more debt makes equity riskier.
- **In fact, a firm's cost of capital is independent of how it finances a project.** Proof follows.
- Assume firm can issue all the risk free debt it wishes.
- We know the following:
  - $r_D = r_f$ (since the debt is risk free.)
  - $\beta_E = \beta_A / e$ (again, the debt is risk free)
  - $r_E = r_f + \beta_E(r_m - r_f)$

Cost of Capital is Independent of Financing: Proof Continued

- Define the total financing cost r_T:

$$r_T = (e)(r_E) + (d)(r_D)$$

- Now substitute out r_E, r_D and use $\beta_E = \beta_A / e$ to get

$$r_T = (e)\left[r_f + \frac{\beta_A}{e}(r_m - r_f) \right] + (d)(r_f).$$

- Finally, with just a bit of algebra:

$$r_T = r_f + \beta_A (r_m - r_f)$$

- Total cost of capital is independent of financing!

Example 1: But Leverage does Raise r_E

- Suppose: $\beta_A = 2$, $A = 100$, $r_f = .05$, $r_m - r_f = .1$
- Let's see how r_E rises as amount of debt, D, rises:

D	β_E	r_E
0	2	.25
10	2.22	.272
50	4	.45
90	20	2.05 (Yes, it really is 2.05!)

Sample Problem with Leveraged Betas

- You can buy the well diversified mutual fund Get Rich Yesterday (GRY) containing only stocks.
  - The mutual fund beta (β_G) = .8. Market portfolio SD = 20%.
  - Your portfolio goal is a standard deviation of 12%.
  - In what proportions should you mix GRY and the risk free asset?
- (Step 1) SD(GRY)=.8(20)=16. (All diversified portfolios with β = .8, must be 80% as risky as the market portfolio.)
- (Step 2) If you combine GRY with the risk free asset:

$$SD(w_G \tilde{r}_G + (1 - w_G) r_f) = w_G \sigma_G = w_G (16) = 12.$$

$$\text{So, } w_G = 3/4 = .75.$$

Sample Problem with Leveraged Betas, Continued

- Now let every firm held by GRY change from 60% equity financing to 40% equity financing.
  - All the debt is risk free
  - What change is required for w_G?
- Recall that $\beta_E=\beta_A/e$, so $\beta_A= e\beta_{E.}$
  - With initial 60% equity financed, firms' $\beta_A = (.6)(.8) =.48$.
  - With final 40% equity financed,$.48 = .4\beta_E$, so $\beta_E = 1.2$.
  - If the new $\beta_E=1.2$, then the S.D. of GRY must be 1.2(20)=24.
  - This implies $w_G(24) = 12$, or $W_G=.5$.
  - So you reduce your holdings in GRY to 50% of your portfolio

Operating Leverage

- An asset creates a portfolio of 3 cash flows:
  - Revenues from the asset (R)
  - Fixed cost to purchase and use the asset (F)
  - Variable cost of using the asset (V)
- Asset cash flow (A) is written as: A = R – F – V
  - Rearrange to get: R = A + F + V.
  - Treat revenues as a portfolio of asset, fixed cost and variable cost.
  - Then use portfolio formula to get revenues beta:

$$\beta_R = \beta_A \frac{A}{R} + \beta_F \frac{F}{R} + \beta_V \frac{V}{R}.$$

Operating Leverage, Continued

- Now start with: $\beta_R = \beta_A \frac{A}{R} + \beta_F \frac{F}{R} + \beta_V \frac{V}{R}.$
  - The beta of the fixed costs equals 0.
  - Also assume revenue beta equals the variable cost beta.
  - So set β_F=0 and β_V=β_R, and we get

$$\beta_R = \beta_A \frac{A}{R} + \beta_R \frac{V}{R}.$$

  - Now solve for $\beta_{A,}$ using R-V = A+F from above:

$$\beta_A = \beta_R \frac{R - V}{A}. \qquad \beta_A = \beta_R \left[1 + \frac{F}{A} \right].$$

Operating Leverage, Conclusion

$$\beta_A = \beta_R \left[1 + \frac{F}{A} \right].$$

- Holding the asset value constant, an increase in the fixed costs increases the asset beta.
- Thus, projects with large fixed costs have their cash flows discounted at a higher rate than projects with low fixed costs.