Lecture 12: Portfolio Theory & CAPM

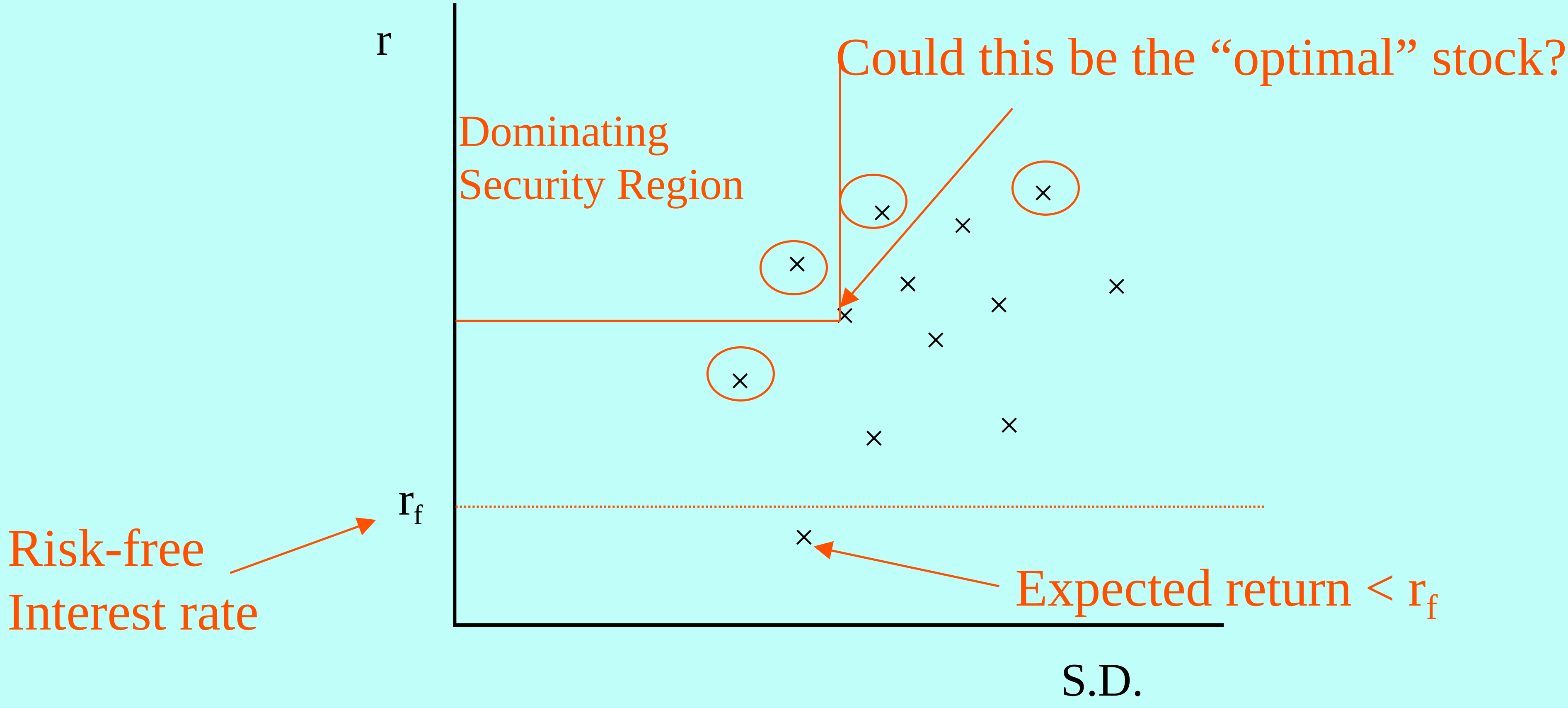
- How are risk and return related?
  - What kinds of risk do we care about?
  - For bearing more risk, how much extra return should we get?

- Brealey and Myers, Chapters 7 and 8
- Reader, Chapters 13 and 14

Variance as a measure of Risk

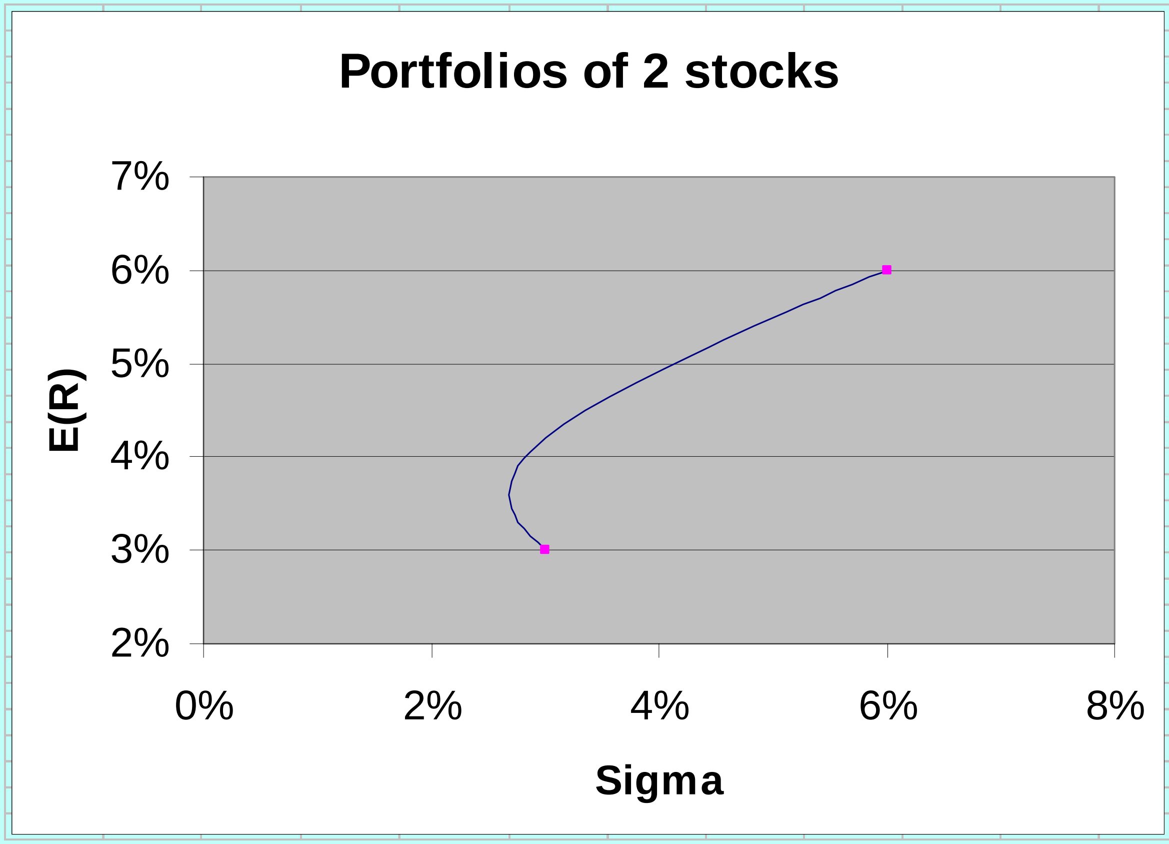
- A person's utility depends only on the **mean** and **variance** of his/her portfolio if <u>either</u> of these assumptions holds:
  - Stock returns are normally distributed; or
  - The utility function is quadratic in return.
- In this case, the **variance** or **standard deviation** of return completely measures the risk of an investment.
- Now consider how an investor should invest
  - The investor faces many securities (stocks) to invest in.
  - Means and variances of each stock are known.

Choices of Possible Securities



- What if we form a **portfolio** of two stocks?

Forming portfolios of 2 stocks

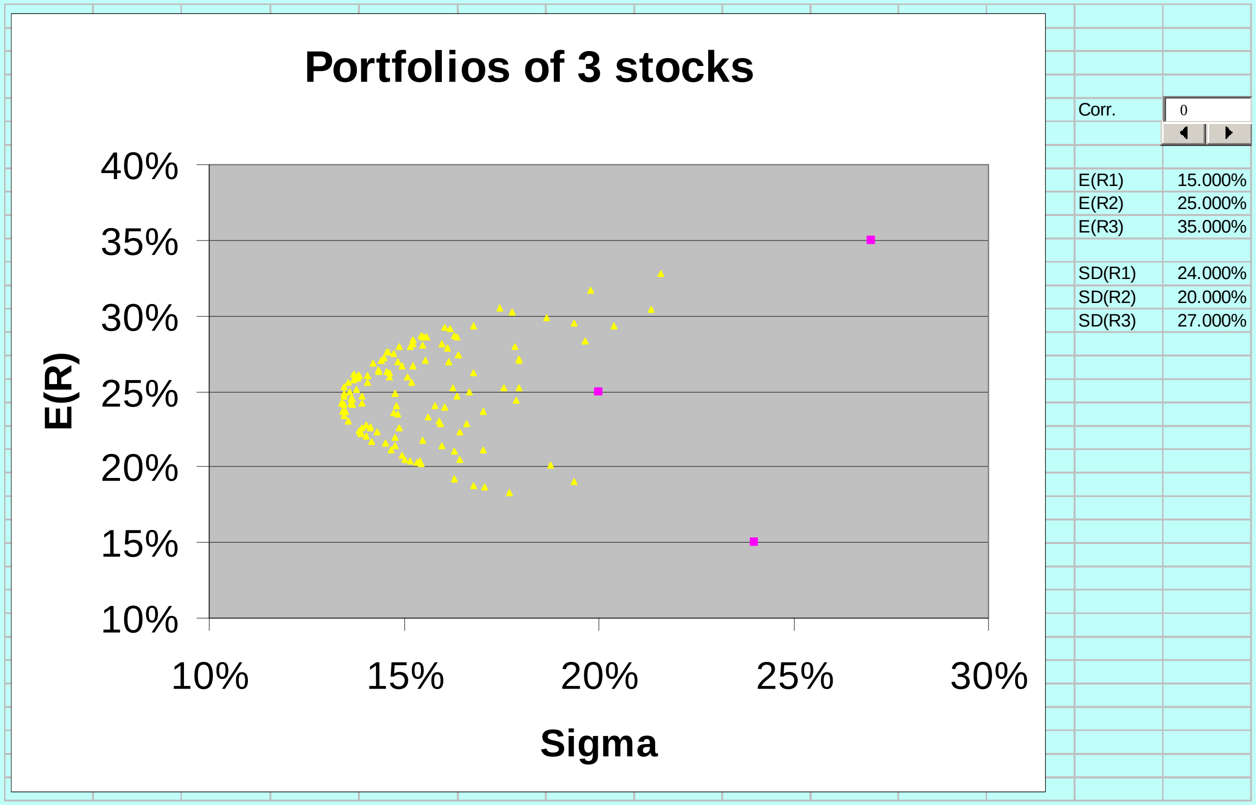


Corr.	0
E(R1)	3.000%
E(R2)	6.000%
SD(R1)	3.000%
SD(R2)	6.000%

w1	SD	r
0	6.00%	6.00%
0.05	5.70%	5.85%
0.1	5.41%	5.70%
0.15	5.12%	5.55%
0.2	4.84%	5.40%
0.25	4.56%	5.25%
0.3	4.30%	5.10%
0.35	4.04%	4.95%
0.4	3.79%	4.80%
0.45	3.57%	4.65%
0.5	3.35%	4.50%
0.55	3.16%	4.35%
0.6	3.00%	4.20%
0.65	2.87%	4.05%
0.7	2.77%	3.90%
0.75	2.70%	3.75%
0.8	2.68%	3.60%
0.85	2.70%	3.45%
0.9	2.77%	3.30%
0.95	2.87%	3.15%
1	3.00%	3.00%

Why does line arc more, the lower the correlation?

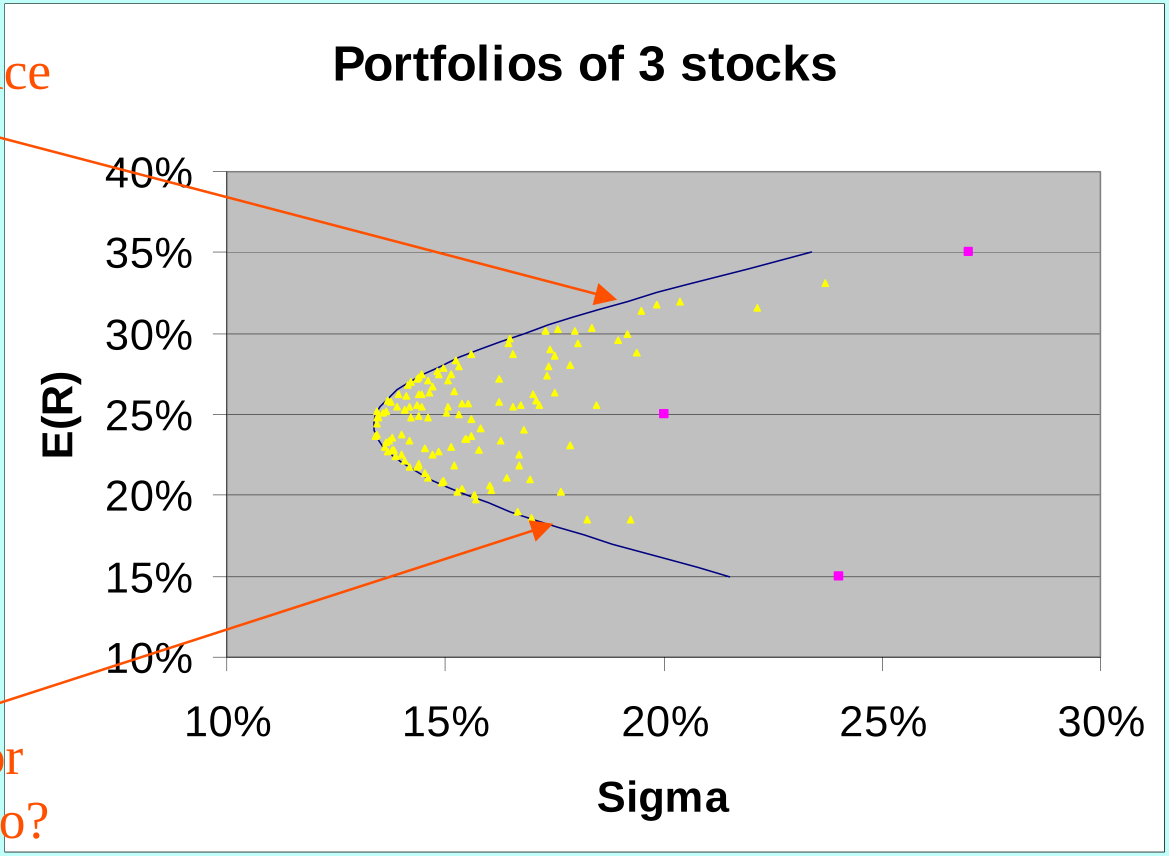
Combining more than 2 stocks



- How far to the left can we go?

Minimum Variance Frontier

Minimum variance frontier



Might an investor pick this portfolio?

Efficient Frontier

- The optimal portfolio for any investor must lie on the top half of the **minimum variance frontier**
  - This is called the **efficient frontier**.
  - No other portfolios offer a higher return for a given SD.
  - Different investors may choose different portfolios.
- How would you find the portfolios on the minimum variance frontier?
  - Minimize variance subject to expected return = r.

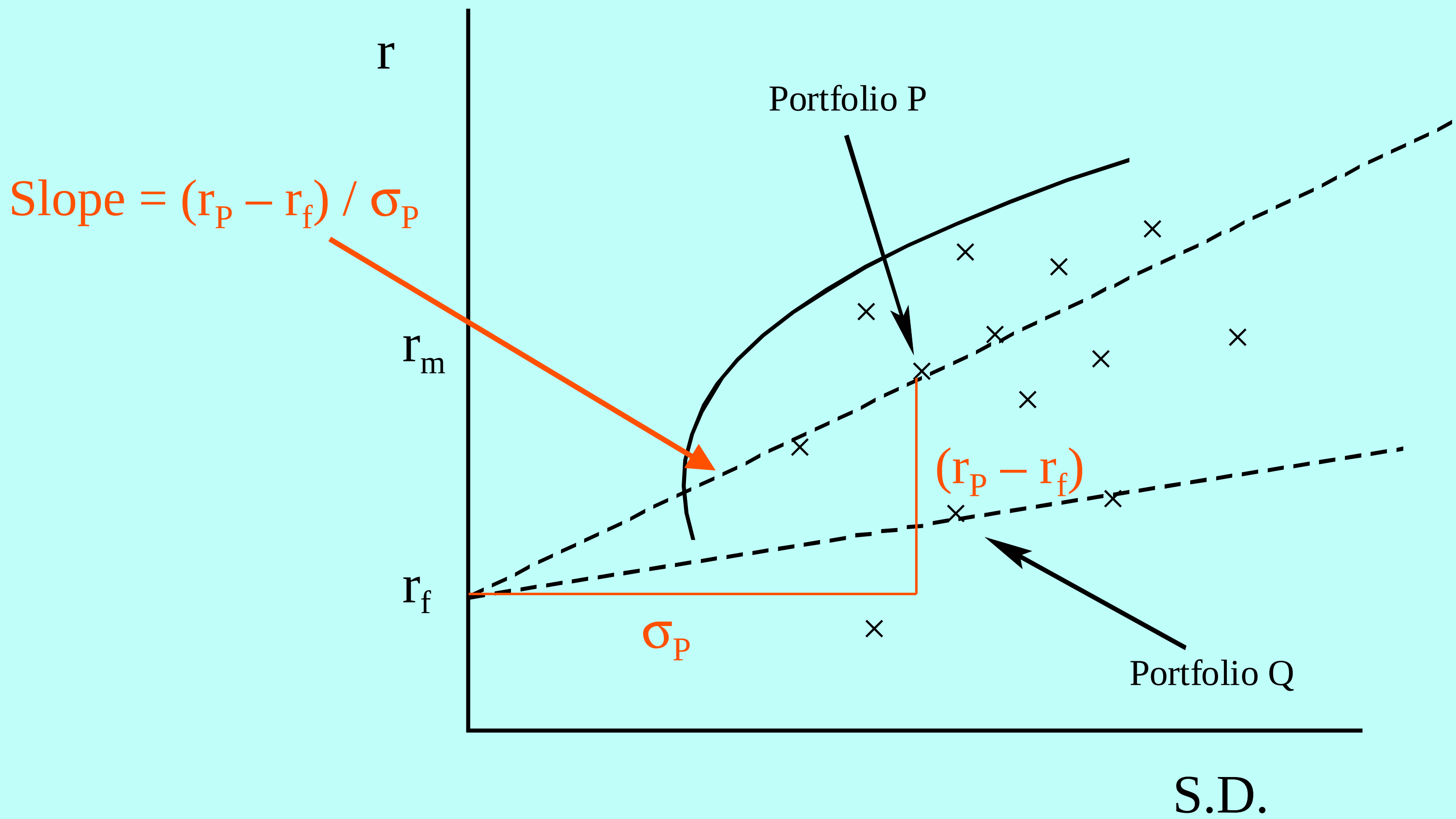
Combining the risk-free asset with a risky portfolio

- Form a portfolio, P, by investing w_s in risky portfolio S, and $(1 - w_s)$ in risk-free asset:
- **Expected return:** $r_P = (1-w_s)r_f + w_s r_s = r_f + w_s(r_s - r_f)$
- **Variance:**

$$\begin{aligned}\mathrm{Var}\left[(1-w_S)r_f + w_S\tilde{r}_S\right] &= (1-w_S)^2 \times 0 + w_S{}^2\mathrm{Var}(\tilde{r}_S) + 2w_S(1-w_S)\times 0 \\ &= w_S{}^2\mathrm{Var}(\tilde{r}_S).\end{aligned}$$

- Or, more compactly, $\sigma_p = w_s\sigma_s$
- Both standard deviation and expected return are **linear** functions of w_S.

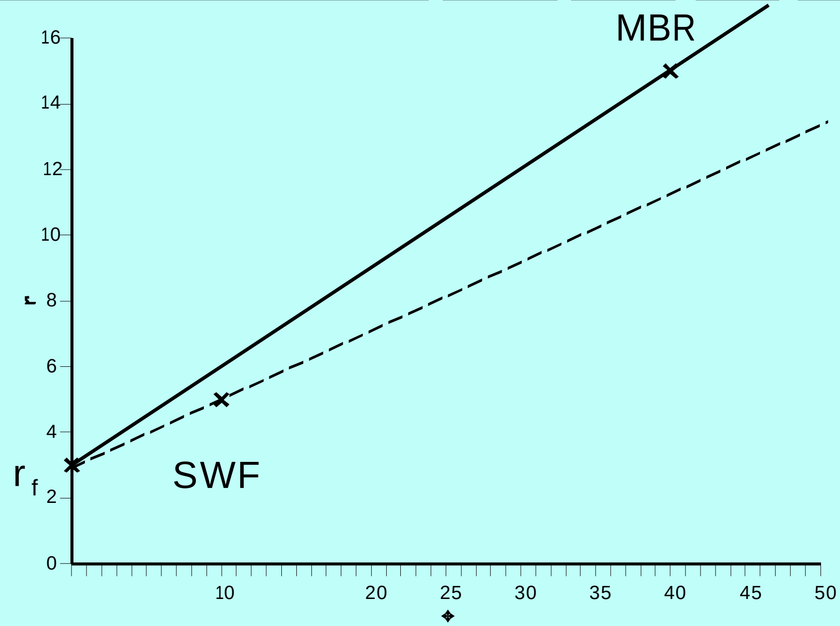
Combining risky portfolios with the riskless asset



Example combining risk-free asset with risky portfolios

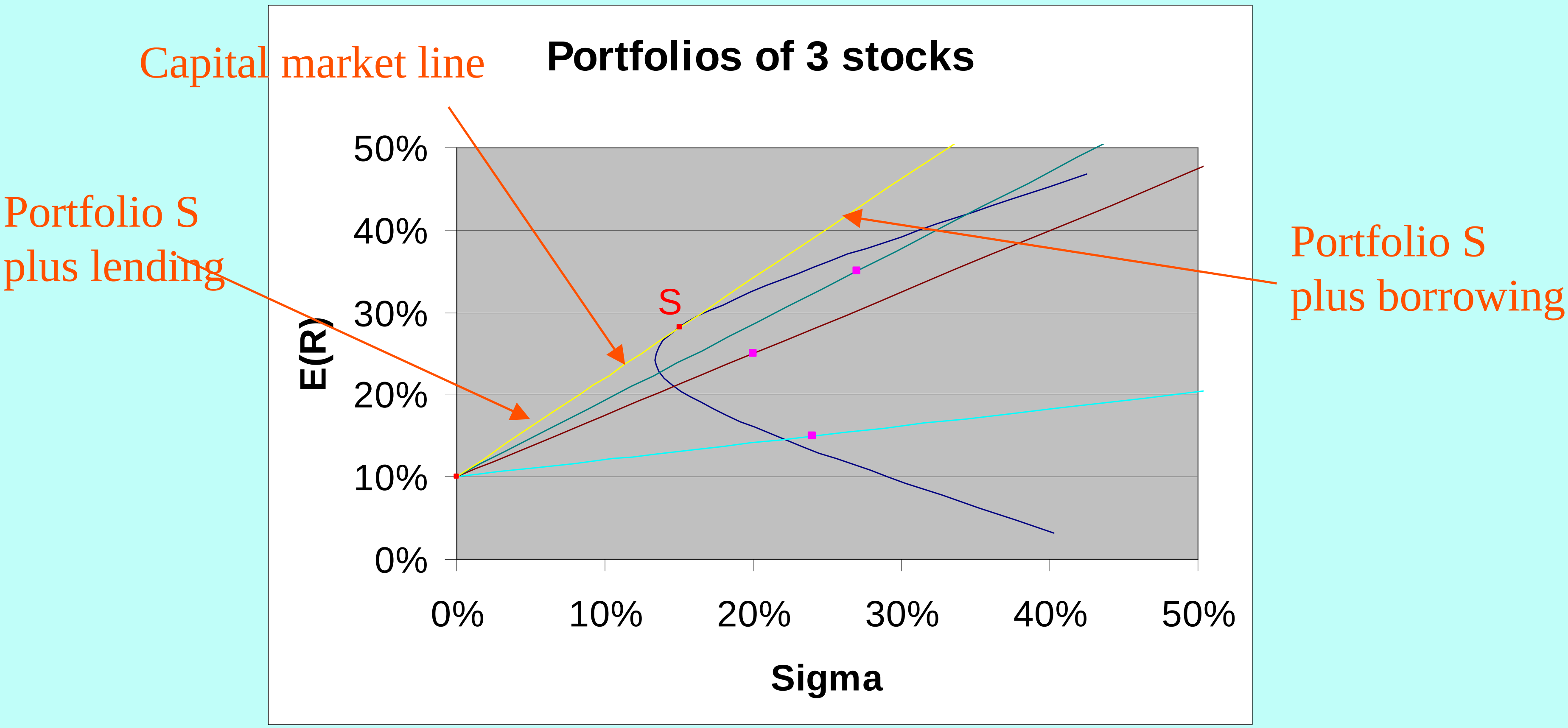
- Mighty Big Returns (MBR) has an expected return of 15%, and a standard deviation of 40%.
- Sleep Well Fund (SWF) has an expected return of 5%, and a standard deviation of 10%.
- The risk-free rate is 3%.
- You can only invest in one of the two funds (e.g. because of minimum purchase requirements)
  - You may also buy the risk-free asset.
- Which fund would you choose?

Solution



- Goal: Find the fund with the higher slope, $(r_s - r_f)/\sigma_s$.
  - This slope (Sharpe ratio) measures extra return for each unit of risk.
- MBR: (15 – 3) / 40 = .3
- SWF: (5 – 3) / 10 = .2

Generally, what's the optimal combination? ($r_f = 10\%$)



The Tangency Portfolio, S

- The optimal portfolio combines S with the risk free asset in some proportions.
- All other portfolios produce a lower line, leading to a lower return for the same standard deviation.
- The line connecting the risk free asset to S is known as the **capital market line**.
- Key point: Every investor chooses the same risky portfolio S.
  - Differ only in the percent allocated to the risk-free asset
- Q: Do all stocks lie on the capital market line?