MGT 890-891
Final Exam Review

Course Organization

MGT 890-891

Module 1
Fundamentals of Valuation

- Types of Security
  - Stock, Bonds and Preferred Stock
  - Cash flows, tax treatment and holder rights
  - Options and derivatives
- Fundamentals of PV
  - How investors and firms make decisions
  - Effect of Borrowing and Lending
  - Interest Rates and Compounding
- Interest Rates
  - Spot Rates, Forward Rates and Bond Prices
  - Bond Price Conventions, Yield to Maturity
  - Real/nominal rates and inflation
- Perpetuities and Annuities
  - Formulae for fixed/growing annuities/perpetuities
  - Loan payments and saving for retirement
- Valuing stocks
  - Price = PV(dividends)
  - Constant dividend growth model
  - P/E ratios, PVGO.
- Alternative Investment Criteria
  - IRR vs. NPV etc.
- Capital Budgeting
  - Making investment decisions
  - The role of taxes

Module 2
Valuing Risky Investments
How Do We Measure Risk?
Relationship Between Risk and Return

- Statistics Review
  - Calculating expected return on a stock or portfolio
  - Calculating variance/covariance of stocks/portfolios
  - Effect of diversification on total risk
- Portfolio Theory
  - Correlation vs. diversification
  - Minimum Variance/Efficient Frontiers
  - Tangency portfolio/Capital Market Line
- Capital Asset Pricing Model (CAPM)
  - Tangency Portfolio = Market Portfolio
  - $r = r_f + \beta (r_m - r_f)$
  - Market Risk => β => Discount Rate
- Effect of Leverage
  - Financial Leverage - Debt vs. Equity
  - Operating Leverage - Fixed vs. Variable Costs
  - Calculating β from cash flows.

Module 3
Corporate Financial Decisions

- Modigliani-Miller *without* Taxes:
  - Pizza size is independent of how sliced
  - Firm value is independent of capital structure
  - Firm value is independent of dividend policy
- Modigliani-Miller *with* Taxes
  - Tax shield of debt
  - Adjusted Present Value (APV)
  - Weighted Average Cost of Capital (WACC)
- Project valuation with APV/WACC (Capstone)
  - Calculate all-equity cash flows
  - Leverage/unleverage betas
  - Discount using rA/WACC, (add S), subtract D

Module 4
Options and Market Efficiency

- Options and Derivatives
  - Derivatives, speculation and hedging
  - Option payoffs
- Option Valuation
  - Factors Affecting Option Value
  - Binomial model, replication and arbitrage
  - Risk neutral probabilities and Black-Scholes
- Market Efficiency
  - Weak, Semi-strong and Strong form efficiency
  - How does information get into prices?

Pre Mid-Term Equations: Summary

- Present Value Equations
  - Present value in terms of spot rates
  - Present value in terms of forward rates
  - Net present value
  - Bond prices and yield to maturity
  - PV of Perpetuity, Growing Perpetuity, and deferred Perpetuity
  - PV of standard and growing annuity

Pre Mid-Term Equations: Summary, Continued

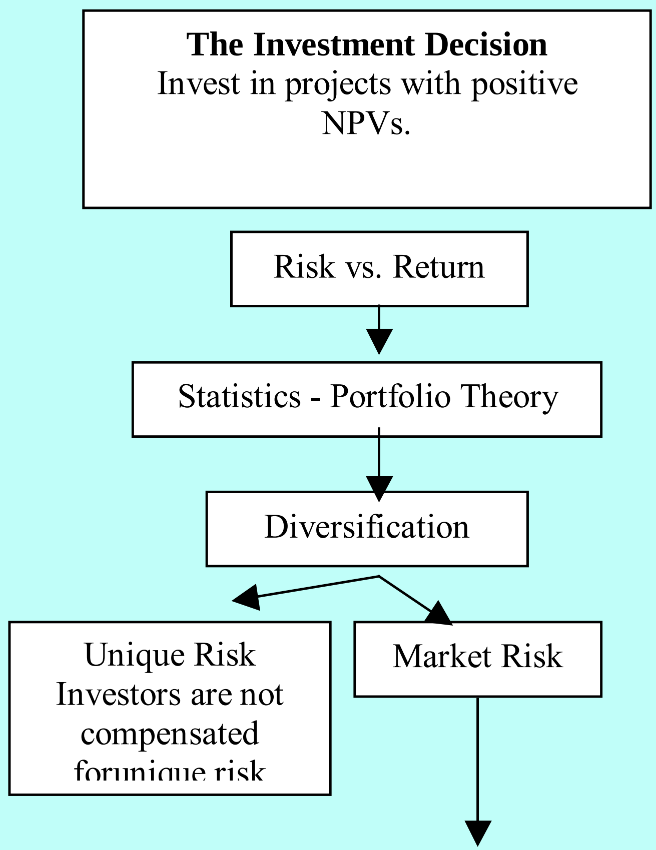
- Conversion equations:
  - between spot rates and forward rates
  - between real and nominal interest rates
  - between real and nominal cash flows
  - between periodic, annual percentage, and annual effective interest rates
- Mortgages/installments:
  - Compute payment, or interest rate, or present value
  - Compute balance outstanding

Pre Mid-Term Equations: Summary, Continued

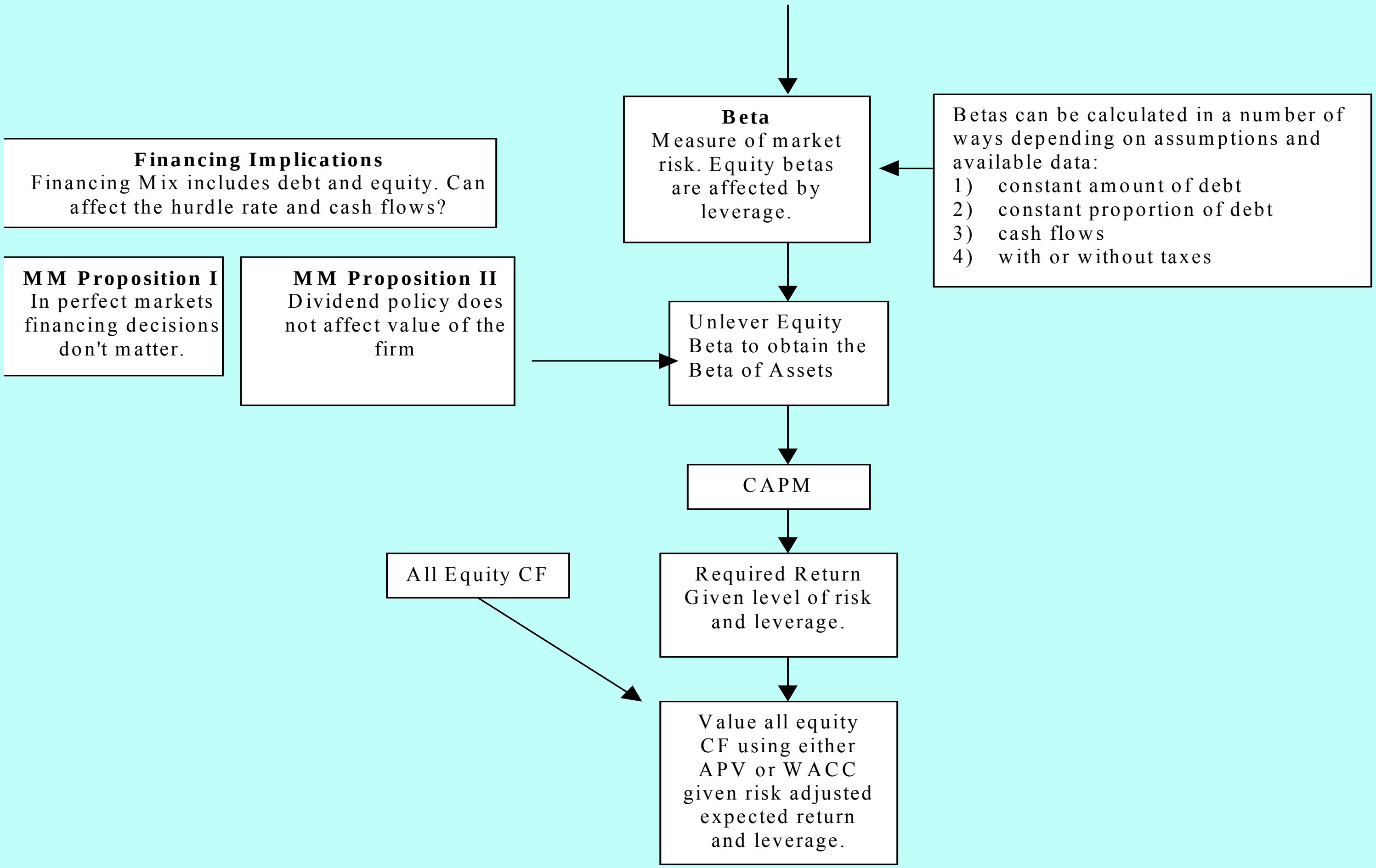
- Simple dividend growth model:
  - Stock price as function of Div, r, and g.
  - Growth rate g as function of plowback and ROE.
  - Present value of growth opportunities
  - Earnings-price ratios

Post Midterm Review 1

CONEPTUAL OVERVIEW



Post Midterm Review 2



A Single Stock: Notation and Formulas

- Notation:
  - $\tilde{r}_i$ is the random return on stock i.
  - r_{ix} is one possible return for stock i, which occurs with probability p_x.
  - r_i is the expected return on stock i.

$$\text{Expected Return} = E(\tilde{r}_i) = r_i = \sum_x p_x r_{ix}$$

$$\text{Variance} = \text{Var}(\tilde{r}_i) = E(\tilde{r}_i - r_i)^2 = \sum_x p_x (r_{ix} - r_i)^2$$

Portfolios: Expected Return

The formula for the expected return r_p on a portfolio:

$$r_p = w_i r_i + w_j r_j$$

- Notation:
  - the expected return on stock i is r_i,
  - the expected return on stock j is r_j,
  - the weight for stock i is w_i,
  - the weight for stock j is w_j.
- Example: $r_i = 3$, $r_j = 5$, $w_i = .4$, and $w_j = .6$. Then: $r_p = (.4)(3) + (.6)(5) = 4.2$.

Covariance: The Relationship between the Return on Stocks

- A zero covariance implies no relationship.
- A positive covariance implies that when stock i has an exceptionally high return, so does stock j.
- A negative covariance implies that when the return on stock i is unusually high, the return on stock j tends to be unusually low.

$$\mathrm{Cov}(\tilde{r}_i, \tilde{r}_j) = E(\tilde{r}_i - r_i)(\tilde{r}_j - r_j) = \quad p_{xy}(r_{ix} - r_i)(r_{jy} - r_j).$$

x y

$$\rho_{ij} = \frac{\mathrm{Cov}(\tilde{r}_i, \tilde{r}_j)}{SD(\tilde{r}_i)SD(\tilde{r}_j)}$$

Rules Governing Portfolio Covariance and Variance

- The covariance for 2 stocks with weights w_i and w_j:

$$\operatorname{cov}\left(w_i \tilde{r}_i, w_j \tilde{r}_j\right) = w_i w_j \operatorname{cov}\left(\tilde{r}_i, \tilde{r}_j\right)$$

- The covariance of a stock with itself:

$$\operatorname{cov}\left(\tilde{r}_i, \tilde{r}_i\right) = \operatorname{var}\left(\tilde{r}_i\right)$$

- The variance of a portfolio with two stocks:

$$\operatorname{var}\left(w_i \tilde{r}_i + w_j \tilde{r}_j\right) = w_i^2 \operatorname{var}\left(\tilde{r}_i\right) + w_j^2 \operatorname{var}\left(\tilde{r}_j\right) + 2 w_i w_j \operatorname{cov}\left(\tilde{r}_i, \tilde{r}_j\right)$$

Multiple Stock Portfolios: Expected Return and Variance

- Expected return on portfolio r_P

$$r_p = w_1 r_1 + w_2 r_2 + ... + w_n r_n$$

Just plug in values and add terms together.

- Variance on portfolio Var_P

$$Var_p = Var\left(w_1 \tilde{r}_1 + w_2 \tilde{r}_2 + ... + w_n \tilde{r}_n\right)$$

Use “box method” to compute portfolio variance.

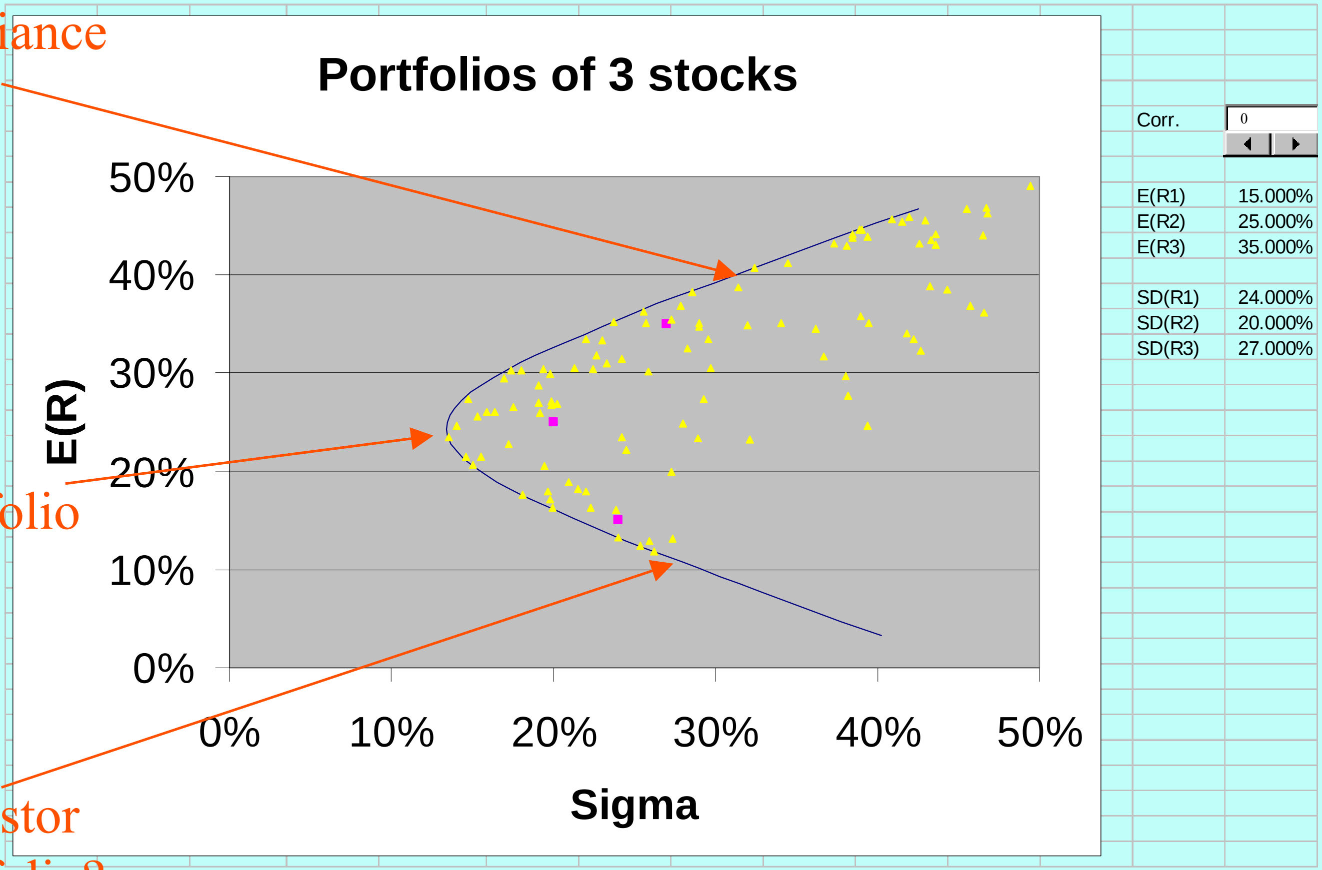
$$\text{Notation}: \sigma_i^2 = \text{variance}_i; \sigma_i = SD_i; \sigma_{ij} = \text{covariance}_{ij}$$

Minimum Variance Frontier

Minimum variance frontier

Minimum variance portfolio

Might an investor pick this portfolio?



Risky and Risk-Free Mixture Portfolio

- Portfolio with w_s in risky portfolio S and $(1 - w_s)$ in risk-free asset:
- The formula for expected return: $r_P = (1\text{-}w_s)r_f + w_s r_s = r_f + w_s(r_s - r_f)$
- Using the variance formula,

$$\mathrm{Var}\left[(1 - w_S)r_f + w_S\tilde{r}_S\right] = (1 - w_S)^2 \times 0 + w_S{}^2\mathrm{Var}(\tilde{r}_S) + 2w_S(1 - w_S) \times 0$$

$= w_S{}^2\mathrm{Var}(\tilde{r}_S)$. Or more compactly, $\sigma_p = (w_s)(\sigma_s)$

- So both standard deviation and expected return for mixture portfolio are **linear** functions of w_S.

Algebraic Statement of The CAPM

Key equations for the CAPM are:

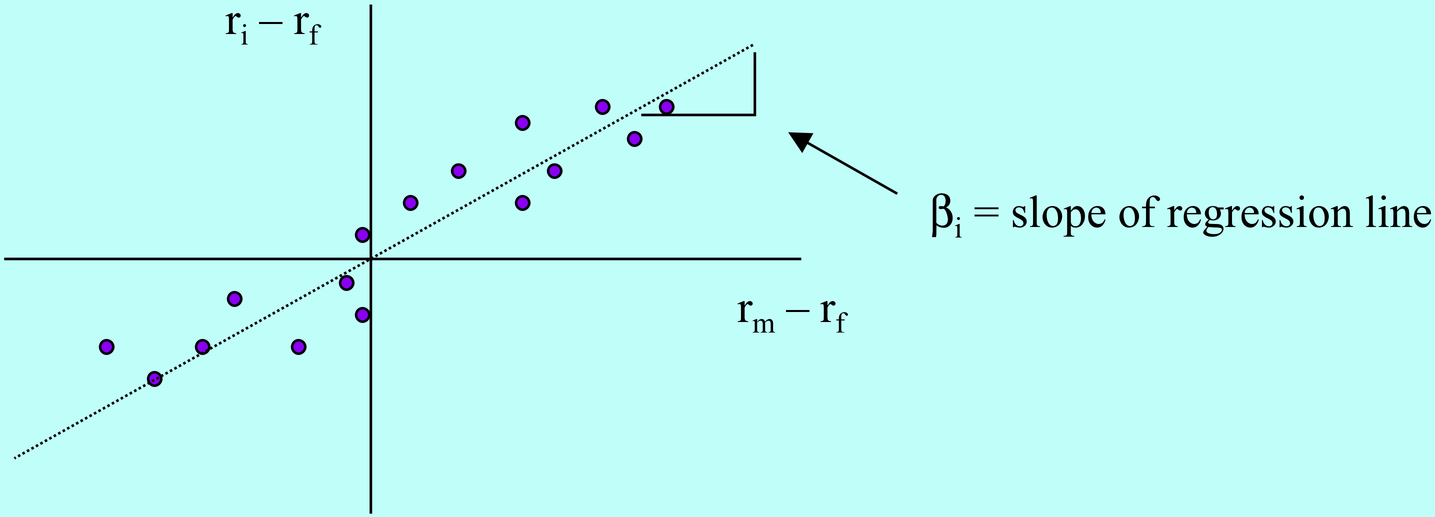
$$r_i = r_f + \beta_i (r_m - r_f)$$

$$\beta_i = \frac{\mathrm{Cov}(\tilde{r}_i, \tilde{r}_m)}{\mathrm{Var}(\tilde{r}_m)}$$

- Expected return is related to the stock's "beta"
  - Depends on <u>covariance</u> with the market: **Market Risk**
  - Does <u>not</u> depend on stock's variance: **Firm Risk**
  - Remember our diversification example…

What does β mean?

- β is defined by $\beta_i = \frac{\mathrm{Cov}(\tilde{r}_i, \tilde{r}_m)}{\mathrm{Var}(\tilde{r}_m)}.$
- This is the definition of a **regression coefficient**.



Beta for any Portfolio

- Consider a portfolio of n securities, with weights $w_1, \ldots, w_n$.
- The beta of the portfolio, β_p, is given by
  $$\beta_p = w_1\beta_1 + w_2\beta_2 + \ldots + w_n\beta_n.$$
- The expected return on the portfolio is
  $$r_p = r_f + \beta_p(r_m - r_f).$$

Interpreting R^2

- The regression also reports an R^2 value
  - 0.32 in our example. What does it tell us?
- The regression can be written as

$$\tilde{r}_i - r_f = \beta_i(\tilde{r}_m - r_f) + \tilde{\varepsilon}.$$

So $$\text{var}(\tilde{r}_i - r_f) = \beta_i^2 \text{var}(\tilde{r}_m - r_f) + \text{var}(\tilde{\varepsilon}).$$

Total risk = Market risk + Firm-specific risk

- R^2 is defined as:

$$R^2 = \frac{\beta_i^2 \text{var}(\tilde{r}_m - r_f)}{\text{var}(\tilde{r}_i - r_f)} = \frac{\text{Market risk}}{\text{Total risk}} = \frac{\text{Market risk}}{\text{Market risk} + \text{Firm-specific risk}}.$$

Two Types of Risk

- Type 1 (Firm specific, or idiosyncratic, risk):
  - Diversification removes firm specific risk from portfolios at no cost.
  - No extra return is earned by holding firm specific risk.
- Type 2 (Market risk):
  - Cannot be eliminated from all portfolios.
  - Thus investors must be paid extra return to hold risk
  - How much? CAPM:

$$\left(r_i - r_f\right) = \beta_i \left(r_m - r_f\right)$$

M&M Proposition 1: Leverage Cannot Influence Firm Value

- Consider two firms, U (unlevered) and L (levered), identical except for capital structure.
  - Firm U is all equity financed, and is worth a total of V_U.
  - Firm L is levered, with equity worth E_L and debt of D_L. Its total value is $V_L = E_L + D_L$.
- Can V_U and V_L be different?
  - No, for exactly the same reason as the pizza.
  - I.e. given either firm, we can create the other our self.

MM Proposition 2: Dividend Irrelevance

- The MM theorem can be extended to show that a firm's dividend policy does not influence its value.
- Our demonstration uses the following example:
  - Initially firm will pay dividend stream such that: Value (equity) = PV(dividends)
  - Now suppose firm raises its dividend at date 0
  - Extra dividend is funded by new debt issued at date 0.
- Key assumption: Changing dividend does not influence underlying net operating income stream!

Converting between Equity and Asset Betas

- Without taxes, the conversion formula is easy:

$$\beta_E = \beta_A\left(1+\frac{D_L}{E_L}\right)-\beta_D\left(\frac{D_L}{E_L}\right).$$

- With taxes, the formula to use depends on assumptions about future debt (see handout for details):

  – Constant amount: $$\beta_E = \beta_A\left[1+\frac{D_L(1-T_C)}{E_L}\right]-\beta_D\left[\frac{D_L(1-T_C)}{E_L}\right].$$

  – Constant proportion: $$\beta_E = \beta_A\left[1+\frac{D_L}{E_L}\left(1-\frac{T_C r_D}{1+r_D}\right)\right]-\beta_D\frac{D_L}{E_L}\left(1-\frac{T_C r_D}{1+r_D}\right).$$

Valuation with Taxes

- There are two methods for valuing a firm (i.e. calculating $V_L = E_L + D_L$ including the value of these tax shields).
- Method 1: **Adjusted Present Value (APV)**
  – Forecast the "all-equity" cash flows (free cash flow + interest paid – interest tax shield).
  – Discount using r_A to get the base PV (if it were all equity financed).
  – Finally, add the PV of all current and future tax shields.
- Method 2: **Weighted Average Cost of Capital (WACC)**
  – Forecast the all-equity cash flows as above
  – Discount using WACC instead of r_A (see handout for explanation)
- Which method you use depends on your assumptions about future debt levels.

Weighted Average Cost of Capital (WACC)

- This is by far the most common method.
- **It only works if two important assumptions hold:**
  - The firm maintains a constant debt / (debt + equity) ratio over its lifetime.
  - The project has the same risk as the rest of the firm.
- The WACC is defined by:

$$\text{WACC} = \frac{D}{D+E}(1-T_c)r_D + \frac{E}{D+E}r_E , \qquad \text{or}$$

$$\text{WACC} = r_A - \left(\frac{D}{D+E}\right)T_C r_D \left(\frac{1+r_A}{1+r_D}\right)$$

where T_C is the firm's tax rate, r_D the firm's borrowing rate.

- Handout shows that these two expressions are equivalent.

Valuation on one side…

Estimate beta
(Market risk)

Regression to estimate β_E

Unlever β_E to obtain β_A, and hence r_A:

Constant debt amount: $\beta_E = \beta_A\left[1+\frac{D_L(1-T_C)}{E_L}\right] - \beta_D\left[\frac{D_L(1-T_C)}{E_L}\right].$

Constant debt proportion: $\beta_E = \beta_A\left[1+\frac{D_L}{E_L}\left(1-\frac{T_C r_D}{1+r_D}\right)\right] - \beta_D\frac{D_L}{E_L}\left(1-\frac{T_C r_D}{1+r_D}\right).$

Forecast **all equity** cash flows

Constant amount of debt

Constant proportion of debt

Calculate WACC from r_A:

$\mathrm{WACC} = r_A - \left(\frac{D_L}{D_L+E_L}\right)T_C r_D\left(\frac{1+r_A}{1+r_D}\right).$

Calculate V_L (= E_L + D_L):
1. Discount all equity CF using r_A.
2. Add PV(tax shield) = T_C x D_L

Calculate V_L (= E_L + D_L):
Discount all equity CF using WACC

Option Terminology

- Strike Price: the price at which the option holder can purchase the stock (or other underlying asset).
- Expiration Date: the final date at which the option can be used.
- Exercising an Option: to use the option to purchase stock.
- Four option positions:
  - Call option buyer or seller
  - Put option buyer or seller
- Market prices are determined by market trading between buyers and sellers for each option contract (theory later).

Factors Affecting Option Value

- The main factors affecting an option's value are:

Factor	**Call**	**Put**
S, stock price	+	-
K, exercise price	-	+
σ, Volatility	+	+
T, expiration date (Am.)	+	+
T, expiration date (Eu.)	+	?
r	+	-
Dividends	-	+

Put-Call parity

- The payoff of the portfolio is identical to that of a European call option.
- The price of the call option must therefore equal the total cost of the portfolio, i.e.

$$C = S + P - K / (1+r)^T$$

- This is called **Put-Call parity**.

Forming a Replicating Portfolio in General

- Form a portfolio today by
  - Buying Δ shares
  - Lending \$B
- Cost today = $\Delta S + B$ (= option price)
- Its value in one year depends on the stock price:

$$\Delta uS + B(1+r)$$

$$\Delta S + B$$

$$\Delta dS + B(1+r)$$

Replicating Portfolio

- We can make the two possible portfolio values equal to the option payoffs by solving:

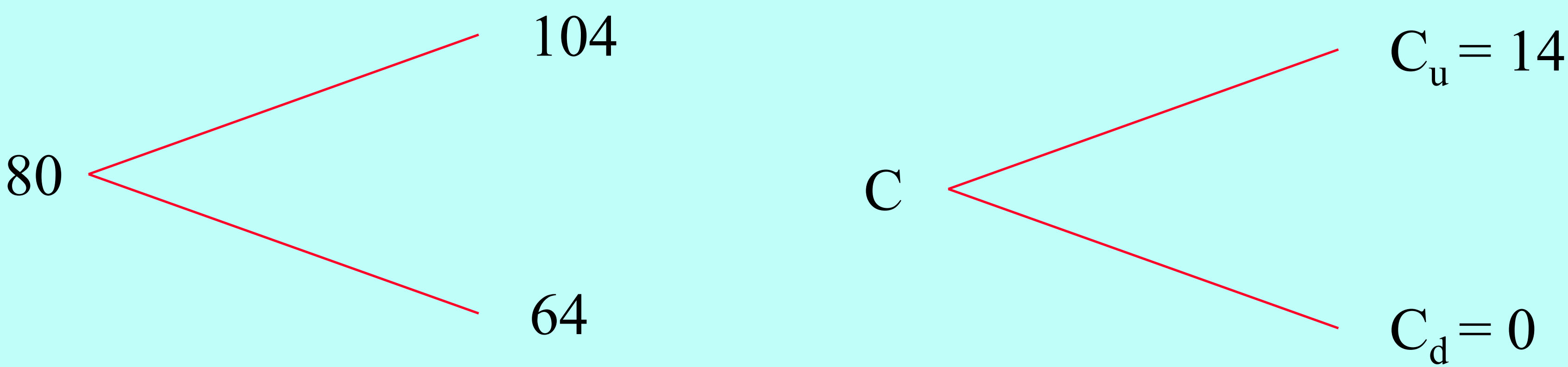
$$\Delta uS + B(1+r) = C_u,$$
$$\Delta dS + B(1+r) = C_d.$$

- Solving these equations, we obtain

$$\Delta = \frac{C_u - C_d}{(u-d)S}, \qquad B = \frac{uC_d - dC_u}{(u-d)(1+r)}.$$

Example

- $S = 80$, $u = 1.3$, $d = 0.8$
- $K = 90$, $r = 10\%$.



Example

- From the formulae,

$$\Delta = \frac{14 - 0}{(1.3 - 0.8)80} = 0.35,$$

$$B = \frac{1.3(0) - 0.8(14)}{(1.3 - 0.8)1.1} = -20.3637.$$

- Hence

$$\begin{aligned} C &= \Delta S + B \\ &= 0.35 \times 80 - 20.3637 \\ &= \$7.6363 \end{aligned}$$

The Efficient Markets Hypothesis

- Market efficiency refers to the extent that market prices reflect all available information.
  - For example, if market prices reflect certain information, then you cannot profit by trading on that information.
- There are 3 primary forms of market efficiency:
  - **Weak form efficiency:** Market prices incorporate all past price information.
  - **Semi-strong form efficiency:** Market prices incorporate all publicly available information.
  - **Strong form efficiency:** market prices incorporate all information.