MGT 890

Bonus Homework Set #2

Answers

# Question 1.

To answer this question use the present value formula for random cash flows

$$PV = \frac{C_1 - \lambda Cov(\tilde{C}_1, \tilde{r}_m)}{1 + r_f}.$$

To begin, calculate the value of the uncertain cash flows from selling the grapes. Since $C_1$ is simply the expected period 1 cash flow from the investment it is easy to calculate in this case. In the bad state the grapes sell for 150 and in the good state 170. So on average you earn .5(150) + .5(170) = 160.

To go any further you need expected return and the variance of the market portfolio. On average the market portfolio pays .5(.05) + .5(.35) = .20. Its variance is therefore $.5(.05-.20)^2+.5(.35-.20)^2 = 0.0225$. Recall $\lambda = (r_m - r_f)/\sigma_m^2$, so it equals (.20-.10)/0.0225 = 4.44 in this case (recall that the problem states that $r_f = 0.10$). The covariance between the cash flows from the grapes and the market portfolio equals .5(150-160)(.05-.20) + .5(170-160)(.35-.20) = 1.50. Plug all of this into the equation to find PV = [160 – (4.44)(1.5)]/(1.10) = 139.40.

Since you only sell the grapes if they makes it back to Earth you only expect to get 80% of the present value calculated above, or 0.8(139.40) = 111.52. Note, we can get the right answer by multiplying like this only because the spaceship's exploding is pure idiosyncratic risk. Otherwise we would need to go through the more complex calculations needed to answer question 2.

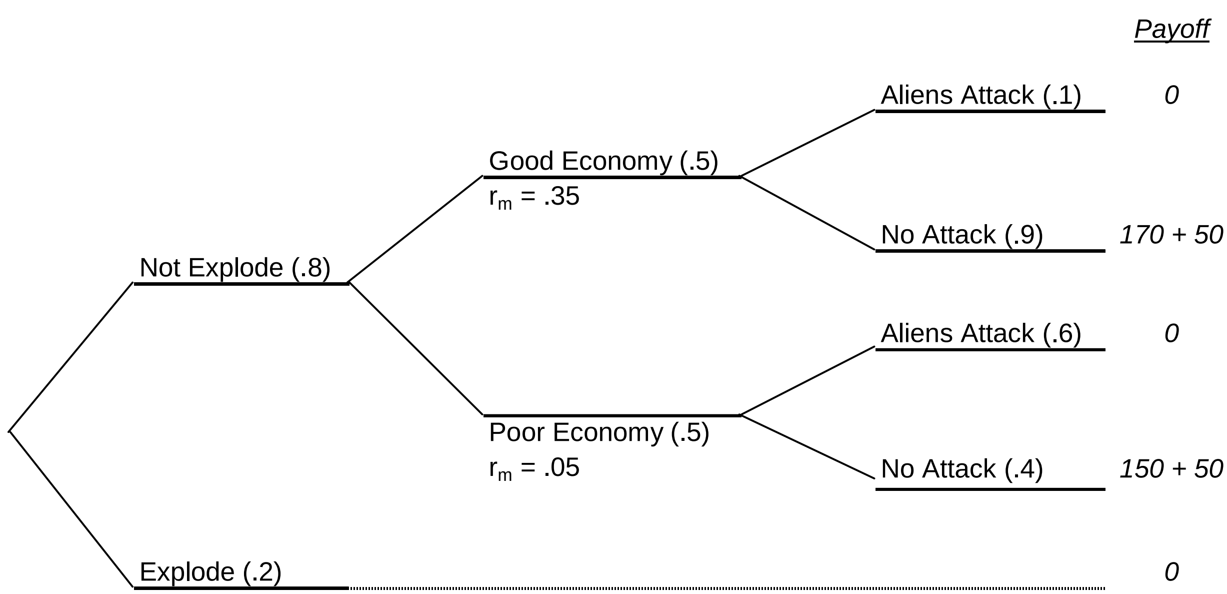
Now for the spaceship. It pays 50 for sure if it makes it back to Earth, and thus has a beta of zero. It therefore costs .8(50)/1.1 - 50 = -13.64. The license and supplies cost 10, so the total profit from this investment equals 111.52-13.64-10 = 87.88.

# Question 2.

Aliens make the problem quite a bit more complicated, since the alienss are more likely to attack in a poor economy than in a good economy. In effect, this ties the probability that the spaceship never returns to Earth to the how well the economy does. I am going to go through two different ways to answer this question. The first follows the problem's language, while the second takes a somewhat more straightforward approach. In both cases the best way to handle this problem is to draw out a tree reflecting all of the possibilities before you try to find the answer.

## Method 1:

The following tree lists the payoff from selling the grapes and the spaceship depending upon what happens to spaceship from either an explosion on its own or an attack by aliens. To reduce the number of calculations we can find the present value of the combined cash flows all at once.




Again, we can do this because the probability that the spaceship explodes on its own is independent from how well the market does.

Step 1: Find the expected cash flow given the spaceship does not explode on its own. With probability (.5)(.9) = .45 the economy will be good and the aliens will not attack. If all this happens you will receive 220.

With probability (.5)(.4) = .20 the economy will be poor and the aliens will not attack. If all this happens you will receive 200.

In all other circumstances the aliens will attack and you will earn nothing. Therefore the expected cash flow (given the spaceship does not explode on its own) equals

$$.45(220) + .20(200) + .35(0) = 139.$$

Step 2: Calculate the covariance between the market and the cash flows.

Recall that the expected return on the market equals .2. Therefore the covariance between the market and the firm's cash flows equals

$$\text{cov}(\tilde{C}_1, \tilde{r}_m) = .45[(220-139)(.35-.20)] + .20[(200-139)(.05-.20)] +$$

$$.05[(0-139)(.35-.20)] + .30[(0-139)(.05-.20)] = 8.85.$$

Step 3: Calculate the present value of the cash flows assuming the ship does not sink from natural causes.

The values of $\lambda$ and the variance of the market were found in problem 1. Using that information plus the covariance calculated above yields

$$PV(\tilde{C}_1) = \frac{139 - (4.44)(8.85)}{1.1} = 90.64$$

Step 4: Calculate the PV of the cash flows.

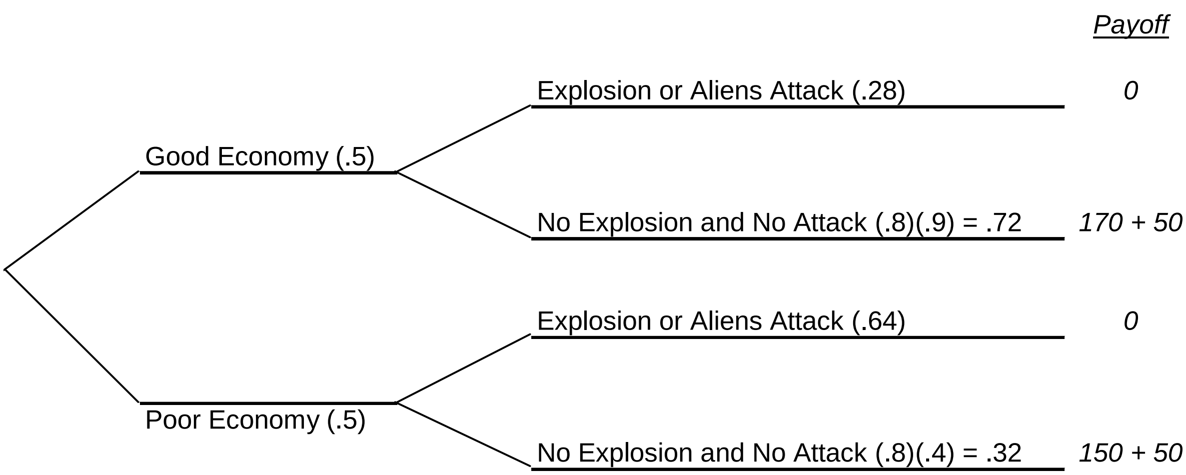
Since the spaceship does not explode on its own with probability .8, the present value of the cash flows from both selling the grapes and the spaceship equals

$$.8(90.64) = 72.51.$$

Recall that it cost 50 for the spaceship, and another 10 for the supplies and licenses for a total of 60. It appears that while the aliens make life somewhat more difficult it still pays to make the investment and harvest the grapes!

## Method 2:

You can reduce the problem's complexity somewhat by noting that you what really matters is the relationship between the payoff and the market. Thus, think of the market as going "first" and the potential explosion in space and the aliens as going second. In this case your payoffs look like this:




Notice that both an explosion and an alien attack produce the same payoffs, zero. Thus, you may as well combine them into a single event. The tree says that when the economy is good the spaceship returns to Earth with probability .72 and you receive 220 in cash. When the economy is bad the spaceship returns to Earth with probability .32 and you receive 200 in cash.

Having already calculated the market statistics from problem 1, we can now answer the question by simply calculating the covariance between the cash flows and the market's return. As always, first you need the expected cash flow:

$$C_1 = (.5)(.72)(220) + (.5)(.32)(200) + (.5)(.28)(0) + (.5)(.64)(0) = 111.20.$$

and now you can calculate the covariance:

$$\text{cov}(\tilde{C}_1, \tilde{r}_m) = (.5)(.72)(220\text{-}111.2)(.35\text{-}.20) + (.5)(.32)(200\text{-}111.20)(.05\text{-}.20)$$

$$+ (.5)(.28)(0\text{-}111.20)(.35\text{-}.05) + (.5)(.68)( 0\text{-}111.20)(.05\text{-}.20) = 7.08.$$

Therefore the present value of the cash flows from making the investment equals

$$PV(\tilde{C}_1) = \frac{111.2 - 4.44 * 7.08}{1.1} = 72.51$$

The same answer as obtained with method 1.

# Question 3.

Given the firm's past revenues their expected value equals

(10+12+6+14+13)/5 = 11.

Since the problem tells you to use a risk free rate of 3% this implies that the market $\lambda$ equals $(.15-.03)/.20^2 = 3.0$.

Using all of the above figures the historical covariance between the firm's revenues and the market equals:

$$\text{Cov}(\text{Rev}, r_m) = (.2-.15)(10-11) + (.08-.15)(12-11) + (-.05-.15)(6-11) + (.17-.15)(14-11) + (.13-.15)(13-11)]/4 = 0.225$$

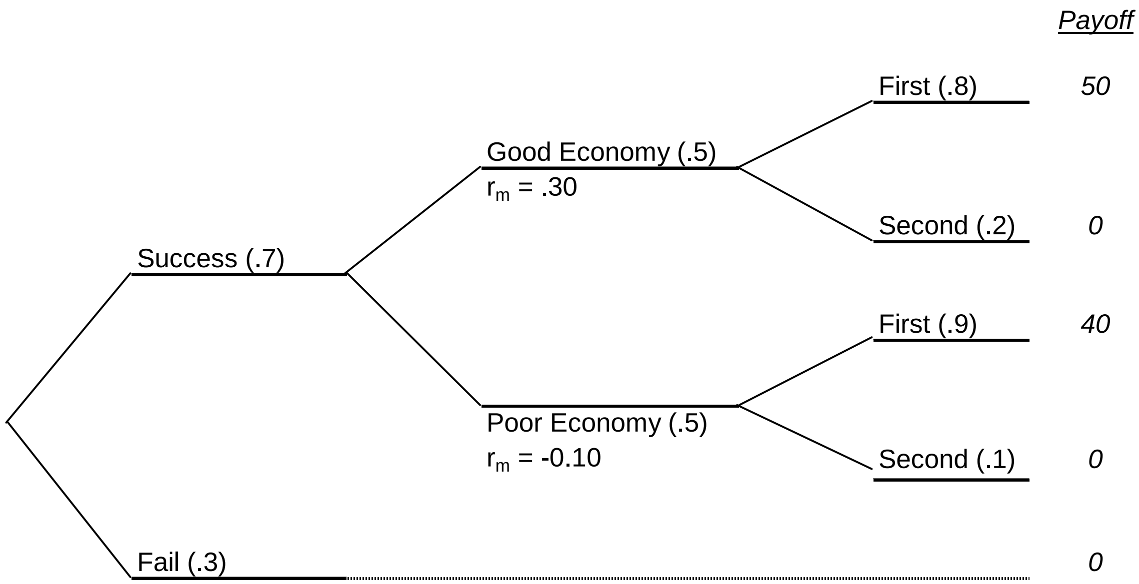
From the formula for the answer to the question is:

$$\beta_{\text{Re}v} = \frac{(1+rf)(Cov(\tilde{C}_1, \tilde{r}_m))}{\sigma_m^2[C_1 - \lambda(Cov(\tilde{C}_1, \tilde{r}_m))]} = \frac{1.03(0.225)}{.20^2\left[11-3(0.225)\right]} = \frac{0.23175}{0.413} = 0.561$$

# Question 4

As with the question 2 it is easier to see what is going on if you put together an event tree.

### Method 1:




As with the question 2 you can reduce the problem's complexity by noting that the success or failure of the research does not depend upon the market. Therefore we can value the project conditional on a successful research effort, and then simply take .7 of that value to get the actual present value of the project.

Step 1: Calculate the market statistics.

The expected return on the market equals .5(.30) + .5(-.10) = .10, and its variance equals $.5(.30-.10)^2 + .5(-.10-.10)^2 = .04$. From these numbers and the fact that the risk free rate is given as .05 the market $\lambda$ equals $(r_m - r_f)/\sigma_m^2 = (.10-.05)/(.04) = 1.25$.

Step 2: Calculate the cash flow statistics conditional on the research effort's success.

Given the research effort succeeds, the expected payoff equals

.5(.8)(50) + .5(.9)(40) + .5(.2)(0) + .5(.1)(0) = 38.

From this the covariance between the project's cash flows and the market equals

$$.5(.8)[(50-38)(.30-.10)] + .5(.9)[(40-38)(-.10-.10)] + .5(.2)[(0-38)(.30-.10)] +$$

$$.5(.1)[(0-38)(-.10-.10)] = 0.40.$$

Therefore the present value conditional on the research effort's success equals

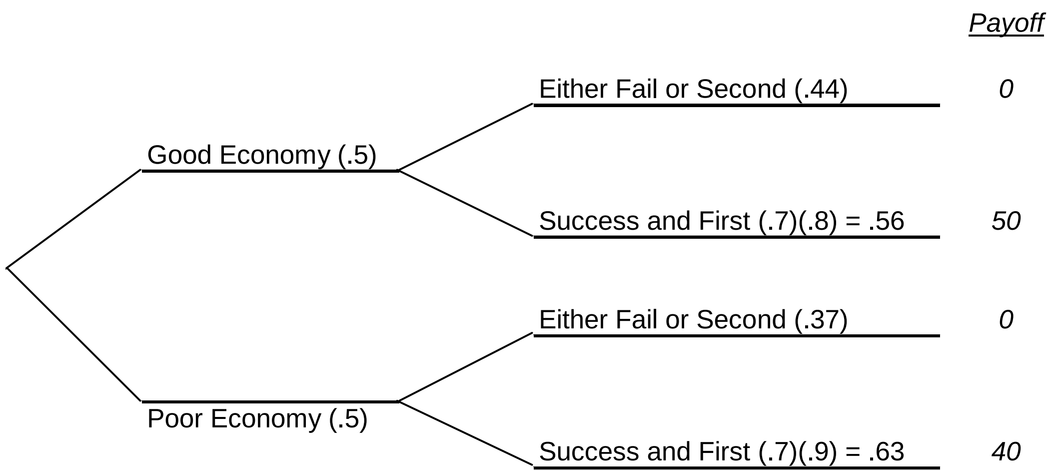
$$PV = \frac{38 - 1.25(.4)}{1.05} = 35.71$$

Step 3: Calculate the unconditional present value.

Since the research effort succeeds only 60% of the time the PV of the cash flows equals (.7)(35.71) = 24.997. Since it costs $20 million to go ahead with the research project you should do it.

## Method 2:

Again the problem's complexity can be reduced by thinking of the market as going first, and then the firm and its competitors.




Based upon the above number the expected cash flow equals:

$$C_1 = .5(.56)(50) + .5(.63)(40) + (.5)(.81)(0) = 26.60$$

and the covariance between the cash flows and the market:

$$\text{cov}(\tilde{C}_1, \tilde{r}_m) = (.5)(.56)(50\text{-}26.6)(.3\text{-}.1) + (.5)(.63)(40\text{-}26.6)(\text{-}.1\text{-}.1)$$

$$+ (.5)(.44)(0\text{-}26.6)(.3\text{-}.1) + (.5)(.37)(0\text{-}26.6)(\text{-}.1\text{-}.1) = 0.28.$$

Therefore the present value of the cash flows equals:

$$PV = \frac{26.6 - (1.25)(0.28)}{1.05} = 25.000$$

which is what was obtained under method 1.