Corporate Finance & Options: MGT 891
Homework #6
Answers

Question 1

A. The APV rule states that the present value of the firm equals it all equity value plus the present value of the tax shield. In this case if the firm uses all equity financing it will keep .6(1000) = 600 per year after taxes. Given a growth rate of .01 this has a value of $600/(r_A-.01)$ where r_A is the discount rate on the cash flow from the firm's assets. The debt tax shield is worth .4(3000)=1200. Thus, one has $600/(r_A-.01) + 1200 = 3000 + 8977.78$. Solving for r_A one finds r_A=.0657. Now use the formula $r_A = r_f + B_A(r_m-r_f)$ to get $.0657 = .05 + B_A(.15-.05)$ or $B_A = .157$.

$B_E = [[E + D(1-t)] / E] * B_A$

$B_E = [[8977.78 + 3000(1-.4)] / 8977.78] *.157 =$ **.188**

B. If we assume that the debt lasts forever then this figure will equal the tax rate times the dollar value of the debt. However, once the firm's debt level depends upon the equity value, and thus the firm's performance, the cash flows from the debt are no longer constant. Instead they depend upon how well the economy does, and thus they now have a positive beta.

We can actually calculate the return to equity in this case, although the problem becomes rather difficult. Listed below are two techniques:

i. Since the debt to equity value is held constant into the future the assumptions needed for the WACC formula hold. The total value of the firm equals 11,977.78. Recall, that the WACC formula states that the total value of the firm equals the *all equity* cash flows discounted at the WACC. In this case the all equity cash flows in period 1 equal .6(1000)=600. Since the firm grows at a rate of 1% per year forever, it can be valued as a perpetuity via the following formula:

$11,977.78 = 600 / (r_{WACC} - .01)$

so $r_{WACC} = .06$. Now use the formula

$r_{WACC} = D/(D+E)(1-T_C)\, r_D + E/(D+E)\, r_E$

$.06 = (3000 / 11,977.78) (.6) (.05) + (8977.78/11,977.78)(r_E)$

which implies $r_E = .0701$.

ii. You can also solve the problem by going back to our original perpetual growth formula. While this requires a bit more care, it does highlight the importance of knowing what assumptions are behind the formulas.

The firm produces a perpetually growing cash flow. As the firm grows, it adjusts the debt to equity ratio so that it remains constant. The constant debt to equity ratio assures you that the return to equity will remain constant over time, and it also ensures the cash flow to equity grows at a constant rate. Therefore, $E = DIV_1/(r_E-g)$. In this case E=8977.78, and g=.01. To get r_E you only need DIV_1. This is not as easy at it sounds. In period 1 the firm will produce a cash flow of 1,000. Of that cash flow .05(3000)=150 will be paid as interest, so the firm will owe .4(1000-150)=340 in taxes. In net, equity gets 1,000-150-340=510 from the original 1,000. However, equity gets another payment. Recall the firm has *grown* 1% since period 0. To keep the debt to equity ratio constant the firm must issue more debt, another .01(3,000)=30 to be exact. What happens to the money raised from the debt issue? Implicitly our financial models require the firm to pay out the extra money as a dividend. Why? Because, we derived our formulas under the assumption that as we changed the firm's financing we left its operations unchanged. The 30 that the firm raises from the debt issue therefore has nowhere to go but to the equity holders. So equity must receive 510 from the 1,000 in profits, and another 30 from the debt issue for a total of 540 in period 1. Plugging this in for DIV_1 produces $8{,}977.78 = 540/(r_E-.01)$ or $r_E=.0701$, which is exactly the same answer that we derived above.

<u>Question 2</u>

This problem shows how flexible the APV method can be.

Part 1: The value of the unlevered equity.

In order to use the APV the first thing you need is the value of the firm under all equity financing. If Salmoneous Shipping were all equity financed it would earn 100,000 in revenues, and pay 60,000 for new ships, and 20,000 in wages. This leaves an operating profit of 20,000 before taxes. According to the problem wages are only 50% deductible as an operating expense. Thus, taxable profits equal

$$100{,}000-60{,}000-.5(20{,}000) = 30{,}000.$$

Of this Salmoneous Shipping will only report 50% of this amount or 15,000, and thus pay taxes of .4(15,000) = 6,000. The shareholders of an all equity firm would therefore collect 20,000-6,000 = 14,000 in year one, and this amount would then grow at a rate of 3%.

The revenues have a beta of .8, and thus this must be the beta for the cash flows that would go to equity if the firm had no debt. Since the problem states that the market has a return of 15% and the risk free rate equals 5%, this leaves a discount rate of .05+.8(.15-.05) = .13. Using the growing perpetuity formula the value of the all equity firm equals 14,000/(.13-.03) = 140,000.

Part II: The PV of the tax shield.

Calculating the value of the tax shield poses a couple of problems. First, the debt is tied to firm's total performance since Salmoneous Shipping tries to keep the debt-equity ratio constant. Second, the adjustment to the debt-equity ratio only takes place every other year and this has to be properly accounted for.

According to the problem in years 1 and 2 the firm will have 90,000 in debt. Since this debt has beta of .1, it must produce a return of .05+.01(.15-.05) = .06 producing interest payments of 5,400 over the next two years. Since the tax rate equals .4, the tax shield from these payments will equal 2,160.

In year 2 Salmoneous Shipping will now issue or retire debt in order to bring its debt-equity ratio back to its original value. As discussed in class this implies the total amount of debt has a beta equal to that of the firm's assets. To see what this does consider the value of the tax shield in years 1, 3, 5, 7, etcetera.

Period 1 Tax Shield: You know that today the firm has 90,000 in debt. This debt will, on average, produce a tax shield of 2,160 in period 1. Since it has a beta of .1 it should be discounted at a rate of 6%.

Period 3 Tax Shield: In period 2, the firm will adjust its debt in order to keep its debt equity ratio constant. Thus today you *expect* the year 3 debt tax shield to grow to $2{,}160 * 1.03^2$. The exact amount of debt that will taken out will of course depend upon how well the firm actually does between now and year two. However, in year two the exact amount of debt issued will be known for sure. Thus, to discount the cash flows from year 3 to year 2 you should use the debt beta of .1, and therefore a discount rate of 6%. Since the year two dollar amount of the debt is tied to the firm's performance you need to use the firm's asset beta and thus a rate of 13% to discount the cash flows from year 2 to year 1, and again from year 1 to year 0.

Period 5+ Tax Shield: Using the same logic we used in the period 3 case, each tax shield should be discounted once at the rate of 6%, and thereafter at a rate of 13%.

PV(Tax Shield Odd Years) = 2,160/1.06 + (2,160∗1.03^2) /(1.06∗1.13^2} + (2,160∗1.03^4)/(1.06∗1.13^4) + …
= 2,037 + 2,037/1.097^2 + 2,037/1.097^4 + …
= 2,037 + 2,037/1.2036 + 2,037/1.2036^2 + …
= 2,037 + 2,037/.2036 = <u>12,046.23</u>

Having solved for the value of the odd year tax shields we now need to solve for the even year tax shields.

Period 2 Tax Shield: Since the firm only issues and retires debt every other year you know for sure that Salmoneous Shipping will have 90,000 in debt in year 1. This means it will have a tax shield of 2,160 in year two. Thus, you can discount this tax shield at the rate of 6% over both years.

Period 4 Tax Shield: In year 2 the firm will alter its debt to restore its debt-equity ratio. You expect the total value of the debt to equal $2{,}160 * 1.03^2$. After that the next adjustment will not take place until year 4. Thus, in year 2 you will know what the firm's tax shield will be in year 4 (as the interest payment depends upon the outstanding debt in year 3). Therefore you can use the

6% rate to bring the year 4 tax shield to year 3, and again to bring it back to year 2. What about from years 2 to 1, and 1 to 0? In year 2 the firm's exact debt level will depend upon its performance, and thus you need to use the asset beta for a discount rate of 13%.

Period 6+ Tax Shield: Using the same logic as above each tax shield should be discount twice at the 6% rate and thereafter at the 13% rate.

PV(Tax Shield Even Years) = 2,160/(1.06^2) + (2,160∗1.03^2)/(1.06^2∗1.13^2) + (2,160∗1.03^4)/(1.06^2∗1.13^4) + …
= 12,046.23/ 1.06 = <u>11,364.36</u>

Therefore the present value of the tax shield equals 12,046.23 + 11,364.36 = 23,410.59. This is quite a bit lower than the 106,442.31 value the analyst calculated and the majority of the difference comes about from the higher discount rate to accommodate the market risk arising from the fixed debt-equity ratio.

<u>To answer the question</u>, the total value of the firm equals 140,000+23,410.59 = 163,410.59. Since the debt has a value of 90,000, this implies the equity is worth 163,410.59-90,000 = 73,410.59. **LESSON**: By using the wrong interest rate the analyst inflated the value of the firm's equity by over 44%! Do not let somebody take you this way.

<u>Question 3</u>

a. You are told the firm's equity has a beta of 1.4, and that the risk free rate equals 5% and the return on the market equals 15%. Therefore the return on equity equals

$$r_E = .05 + 1.4(.15-.05) = \mathbf{.19}.$$

b. Since the firm has a debt-equity ratio of 1:3 its WACC equals

r_{WACC} = D/(D+E) (1-T_c)(r_D) + E/(D+E) (r_E)
r_{WACC} = (D/E)/(D/E+1) (1-T_c) r_D + 1/(D/E+1) (r_E)
r_{WACC} = (1/3)/(1/3+1) (1-.4).06 + 1/(1/3+1) (.19)
r_{WACC} = **.1515**.

c. 10/.1515 = **66 million**.

d. In this case you would need to use some other technique like the APV. You cannot use the WACC since the risk of the new project is not like that of the firm's other operations and thus the WACC will not produce the proper discount rate.

<u>Question 4</u>

We know from MM Proposition I that value of the firm will not change with leverage. This implies that B_A and r_A will remain the same for all debt levels. However, additional debt implies additional risk as reflected in the increasing equity beta with increasing proportion of debt (see

table).
In the case were debt is risk free, B_D equals zero for all levels of W_D. The result is much higher equity betas (B_S) due to proportional risk coming all from equity. Also, the implication is that return on equity must increase further when debt is risk free since cost of capital (r_A) remains constant. See table below for results.

Solve for B_S: $B_A = [D/(D+E)]\ B_D + [E/(D+E)]\ B_S$

B_A	W_D	B_D	W_S	B_S
		Varying B_D		
0.75	0.00	0.00	1.00	**0.75**
0.75	0.25	0.13	0.75	**0.96**
0.75	0.50	0.25	0.50	**1.25**
0.75	0.75	0.38	0.25	**1.88**
		Debt Risk Free		
0.75	0.00	0.00	1.00	**0.75**
0.75	0.25	0.00	0.75	**1.00**
0.75	0.50	0.00	0.50	**1.50**
0.75	0.75	0.00	0.25	**3.00**

Question 5

A) Given, $B_A = [D/(D+E)]\ B_D + [E/(D+E)]\ B_S = .9$

$.9 = (.4)(0) + (.6)\ B_S$
$B_S = 1.5$

B) A = Asset Value, R = Revenue, F = Fixed Cost of Asset use, V = Variable Cost of asst use, and A = R - F - V.

Initially,
A = 50, R =100, V = 40, F =10, and $B_A = .9$

If $B_F = 0$ and $B_V = B_R$, then $B_R = B_A\ (A/R) + B_R\ (V/R)$ or $B_A = B_R\ (R-V)/A$
$B_A = .9 = B_R\ (100-40)/50$
$B_R = .75$

New,
A = 60, R = (100 + 40), V = (40 + 20) = 60, F = (10 + 10) = 20
$B_A = .75\ (140-60)/60$
$B_A = 1.00$

C) $1.0 = (.4)(0) + (.6)\ B_S$
$B_S =$ **1.667**