

Pricing Behavior in Markets with State Dependence in Demand

Technical Appendix

(for review only, not for publication)

This Draft: July 5, 2006

Introduction

In this technical appendix, we provide additional information that we did not include in the main paper to keep the paper's length reasonable. The technical appendix consists of six sections:

Section 1 provides the extension of the 2 period supply model shown in the paper to *a general T period model*.

Section 2 *generalizes the key research findings of the paper to the Ketchup category*, in order to assess the robustness of the empirical findings.

Section 3 estimates the demand model for the cereal category with *continuous heterogeneity* and compares the elasticity estimates with the discrete heterogeneity specification. The elasticity estimates are similar suggesting that the use of a discrete heterogeneity specification in the paper is reasonable.

Sections 4- 6 report several simulation results to assess the robustness of the findings.

Section 4 shows that the estimator "works" by estimating the demand model with simulated data that is identical in structure and estimates to the actual data.

Section 5 demonstrates using simulated data that the key supply side empirical findings of the paper are robust for a range of parameter values (demand intercepts, state dependence, levels of forward looking behavior).

Section 6 addresses the concern that it is possible that firms may have a longer horizon (one quarter look ahead) than the weekly look-ahead that we tested in our empirical analysis. We find that indeed if the data are generated based on a one quarter look ahead, the estimator will detect such behavior, suggesting that our results are not due to a different time horizon used by firms.

1. A Finite T -Period Look Ahead Supply Side Model

Suppose that each manufacturer and the retailer maximizes T -period inter-temporal profits. We show the derivation first using a three-period model, which is then generalized to the T -period model.

1. Two-period Look Ahead (Three Period) Pricing Model

The Manufacturer

Suppose there are F firms, each of which produces some subset, F_f , of the $j = 1, \dots, J$ different brands in the product category. The profits in each period for manufacturer f are

$$\pi_{f_t} = \sum_{r \in F_f} (w_{jt} - mc_j) M s_{jt}(p) \quad (1)$$

where $t = 1, 2, 3$. $s_j(p)$ is the market share of brand j , which is a function of the prices of all brands, M is the size of the market.

If manufacturers maximize three-period profits, the objective function will become

$$V_f = \pi_{f_1}(w_{j_1}) + \delta \pi_{f_2}(w_{j_2}) + \delta^2 \pi_{f_3}(w_{j_3}) \quad (2)$$

where $j \in F_f$.

Assuming the existence of a pure-strategy **Bertrand-Nash** equilibrium in prices, and that the prices that support it are strictly positive, the wholesale price w_{jt} of any brand j produced by firm f must satisfy the following first-order condition.

$$\begin{aligned} \frac{\partial \pi_{f_1}}{\partial w_{j_1}} + \delta \sum_{r \in F_f} \frac{\partial \pi_{f_2}}{\partial s_{r_2}(p)} \frac{\partial s_{r_2}(p)}{\partial s_{r_1}(p)} \frac{\partial s_{r_1}(p)}{\partial p_{j_1}} \frac{\partial p_{j_1}}{\partial w_{j_1}} + \delta^2 \sum_{r \in F_f} \frac{\partial \pi_{f_3}}{\partial s_{r_3}(p)} \frac{\partial s_{r_3}(p)}{\partial s_{r_2}(p)} \frac{\partial s_{r_2}(p)}{\partial s_{r_1}(p)} \frac{\partial s_{r_1}(p)}{\partial p_{j_1}} \frac{\partial p_{j_1}}{\partial w_{j_1}} &= 0 \\ \frac{\partial \pi_{f_2}}{\partial w_{j_2}} + \delta \sum_{r \in F_f} \frac{\partial \pi_{f_3}}{\partial s_{r_3}(p)} \frac{\partial s_{r_3}(p)}{\partial s_{r_2}(p)} \frac{\partial s_{r_2}(p)}{\partial p_{j_2}} \frac{\partial p_{j_2}}{\partial w_{j_2}} &= 0 \\ \frac{\partial \pi_{f_3}}{\partial w_{j_3}} &= 0 \end{aligned} \quad (3)$$

In detail,

$$\frac{\partial \pi_{jt}}{\partial w_{jt}} = \left[s_{jt}(p) + \sum_{r \in F_f} (w_{rt} - mc_r) \frac{\partial s_{rt}(p)}{\partial p_{jt}} \frac{\partial p_{jt}}{\partial w_{jt}} \right] M \quad (4)$$

where $t = 1, 2, 3$. Compared to the static model case, there are four new terms in the first order conditions, $\frac{\partial \pi_{r2}}{\partial s_{r2}(p)}$, $\frac{\partial s_{r2}(p)}{\partial s_{r1}(p)}$, $\frac{\partial s_{r3}(p)}{\partial s_{r2}(p)}$ and $\frac{\partial \pi_{f3}}{\partial s_{r3}(p)}$, we assume the following relationship holds between s_{r1} and s_{r2} , and also between s_{r2} and s_{r3} .

$$s_{r(t+1)} = \theta_{r(t+1) \cdot r} * s_{rt} + \sum_{k=1, k \neq r}^J \theta_{r(t+1) \cdot kt} * s_{kt}$$

where, as defined in the demand models, $\theta_{r(t+1) \cdot kt}$ is the transition probability that consumers who **bought** brand r in period t and **continue to buy** it during period $t+1$ and $\theta_{r(t+1) \cdot kt}$ is the transition probability that consumers who **did not buy** brand r in period t **switch to it** during period $t+1$.

It can be shown that

$$\frac{\partial s_{r(t+1)}(p)}{\partial s_{rt}(p)} = \theta_{r(t+1) \cdot r} - \sum_{k=1, k \neq r}^J \theta_{r(t+1) \cdot kt} = \begin{cases} > 0 & \text{if } SD > 0 \text{ and } \theta_{r(t+1) \cdot r} > \sum_{k=1, k \neq r}^J \theta_{r(t+1) \cdot kt} \\ < 0 & \text{if } SD < 0 \text{ and } \theta_{r(t+1) \cdot r} < \sum_{k=1, k \neq r}^J \theta_{r(t+1) \cdot kt} \end{cases}$$

Therefore

$$\begin{aligned} \frac{\partial \pi_{f(t+1)}}{\partial s_{r(t+1)}(p)} &= (w_{r(t+1)} - mc_r) \\ \frac{\partial s_{r(t+1)}(p)}{\partial s_{rt}(p)} &= \Delta_t = \bar{\theta}_{r(t+1) \cdot r} - \sum_{k=1, k \neq r}^J \bar{\theta}_{r(t+1) \cdot kt} \end{aligned} \quad (5)$$

where $t = 2, 3$.

The detailed first order conditions then become

$$\begin{aligned} s_{j1}(p) + \sum_{r \in F_f} (w_{r1} - mc_r) \frac{\partial s_{r1}(p)}{\partial p_{j1}} \frac{\partial p_{j1}}{\partial w_{j1}} + \delta \sum_{r \in F_f} (w_{2r} - mc_r) \Delta_1 \frac{\partial s_{r1}(p)}{\partial p_{j1}} \frac{\partial p_{j1}}{\partial w_{j1}} \\ + \delta^2 \sum_{r \in F_f} (w_{3r} - mc_r) \Delta_2 \Delta_1 \frac{\partial s_{r1}(p)}{\partial p_{j1}} \frac{\partial p_{j1}}{\partial w_{j1}} = 0 \end{aligned} \quad (6)$$

$$s_{j2}(p) + \sum_{r \in F_f} (w_{r2} - mc_r) \frac{\partial s_{r2}(p)}{\partial p_{j2}} \frac{\partial p_{j2}}{\partial w_{j2}} + \delta \sum_{r \in F_f} (w_{3r} - mc_r) \Delta_2 \frac{\partial s_{r2}(p)}{\partial p_{j2}} \frac{\partial p_{j2}}{\partial w_{j2}} = 0 \quad (7)$$

$$s_{j3}(p) + \sum_{r \in F_f} (w_{r3} - mc_r) \frac{\partial s_{r3}(p)}{\partial p_{j3}} \frac{\partial p_{j3}}{\partial w_{j3}} = 0 \quad (8)$$

The manufacturer's profit function is a set of $J \times 2$ equations, each of them implies price-costs margins for each brand. The markups can be solved for explicitly by defining $\Phi_{jr} = -\frac{\partial s_r(p)}{\partial p_j}$, $j, r = 1, \dots, J$,

$$\Omega_{jr}^* = \begin{cases} 1, & \text{if } \exists f : \{r, j\} \subset F_f \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

and Ω is a $J \times J$ matrix with $\Omega_{jrt} = \Omega_{jr}^* \times \Phi_{jrt}$.

In vector notation, the first-order conditions for period 3 become

$$s_3(p) - \Omega_3(w_3 - mc) = 0 \quad (10)$$

where $s(\cdot)$, p and mc are $I \times 1$ vectors of market shares, prices and marginal-cost, respectively. This implies a mark-up equation,

$$\begin{aligned} PCM^{Intertemporal}(t=3) &= w_3 - mc \\ &= \Omega_3^{-1} s_3(p) \\ &= PCM^{Myopic}(t=3) \end{aligned} \quad (11)$$

Therefore, **using estimates from the demand side**, we could estimate PCM without actually observing actual costs.

The second-period first-order conditions are

$$\begin{aligned} s_2(p) - \Omega_2(w_2 - mc) - \Omega_2[\delta(w_{r3} - mc_r)\Delta_2] \\ = s_2(p) - \Omega_2(w_2 - mc) - \Omega_2[\delta\Omega_3^{-1}s_3(p)\Delta_2] \\ = 0 \end{aligned} \quad (12)$$

which is equivalent to

$$\begin{aligned} PCM^{Intertemporal}(t=2) &= w_2 - mc_2 \\ &= \Omega_2^{-1} \{s_2(p) - \Omega_2[PCM(t=3)\Delta_2]\} \\ &= PCM^{Myopic}(t=2) - \delta\Delta_2 PCM^{Intertemporal}(t=3) \end{aligned} \quad (13)$$

Lastly

$$s_1(p) - \Omega_1(w_1 - mc) - \Omega_1[\delta(w_{r2} - mc_r)\Delta_1] - \Omega_1[\delta^2(w_{r3} - mc_r)\Delta_1\Delta_2] = 0 \quad (14)$$

and, therefore,

$$\begin{aligned} PCM^{Intertemporal}(t=1) &= w_1 - mc \\ &= \Omega_1^{-1}s_1(p) - \delta\Omega_2^{-1}s_2(p)\Delta_1 \\ &= PCM^{Myopic}(t=1) - \delta\Delta_1PCM^{Intertemporal}(t=2) \end{aligned} \quad (15)$$

The retailer

The retailer takes the wholesale prices as given, and acts a monopolist in pricing the whole category. The retailer's problem is thus as shown below.

$$\max_{p_1, \dots, p_J} \pi_{Rt} = \sum_{j=1}^J (p_{jt} - w_{jt})s_{jt}(p) \quad (16)$$

where $t=1,2$. $s_j(p)$ is the market share of brand j , which is a function of the prices of all brands, M is the size of the market.

If the monopolist retailer also maximizes three-period profits, his objective function will become

$$V_R = \pi_{R1}(w_{j1}) + \delta\pi_{R2}(w_{j2}) + \delta^2\pi_{R3}(w_{j3}) \quad (17)$$

where $j=1, \dots, J$.

Analogous to the manufacturers' case, the detailed first-order conditions are

$$\begin{aligned} s_{j1}(p) + \sum_{j=1}^J (p_{r1} - w_r) \frac{\partial s_{r1}(p)}{\partial p_{j1}} + \delta \sum_{j=1}^J (p_{2r} - w_r) \Delta_1 \frac{\partial s_{r1}(p)}{\partial p_{j1}} \\ + \delta^2 \sum_{j=1}^J (p_{3r} - w_r) \Delta_2 \Delta_1 \frac{\partial s_{r1}(p)}{\partial p_{j1}} = 0 \end{aligned} \quad (18)$$

$$s_{j2}(p) + \sum_{j=1}^J (p_{r2} - w_r) \frac{\partial s_{r2}(p)}{\partial p_{j2}} + \delta \sum_{j=1}^J (p_{3r} - w_r) \Delta_2 \frac{\partial s_{r2}(p)}{\partial p_{j2}} = 0 \quad (19)$$

$$s_{j3}(p) + \sum_{j=1}^J (p_{r3} - w_r) \frac{\partial s_{r3}(p)}{\partial p_{j3}} = 0 \quad (20)$$

and everything else in the above two equations is defined as in the manufacturer's case.

Defining $\Phi_{jrt} = -\frac{\partial s_{jt}(p)}{\partial p_r}$, $j, r = 1, \dots, J$, and using the matrix form, the retailer's third-period first-order condition could be written as

$$\begin{aligned} PCM^{Intertemporal}(t=3) &= p_3 - mc \\ &= \Phi_3^{-1} s_3(p) \\ &= PCM^{Myopic}(t=3) \end{aligned} \quad (21)$$

The second-period first-order conditions are given below.

$$\begin{aligned} s_2(p) - \Phi_2(p_2 - mc) - \Phi_2[\delta(p_{r3} - mc_r)\Delta_2] \\ = s_2(p) - \Phi_2(p_2 - mc) - \Phi_2[\delta\Phi_3^{-1}s_3(p)\Delta_2] \\ = 0 \end{aligned} \quad (22)$$

which is equivalent to

$$\begin{aligned} PCM^{Intertemporal}(t=2) \\ = p_2 - w_2 \\ = PCM^{Myopic}(t=2) - \delta\Delta_2 PCM^{Intertemporal}(t=3) \end{aligned} \quad (23)$$

Lastly

$$s_1(p) - \Phi_1(p_1 - w) - \Phi_1[\delta(p_{r2} - w_r)\Delta_1] - \Phi_1[\delta^2(p_{r3} - w_r)\Delta_1\Delta_2] = 0 \quad (24)$$

and, therefore,

$$\begin{aligned} PCM^{Intertemporal}(t=1) \\ = p_1 - w \\ = PCM^{Myopic}(t=1) - \delta\Delta_1 PCM^{Intertemporal}(t=2) \end{aligned} \quad (25)$$

Price-Cost Margin in the Three-period Model

The price-cost margin is defined as

$$\begin{aligned} p_t - mc_t &= (p_t - w_t) + (w_t - mc_t) \\ &= \begin{cases} PCM^{myopic}(t=1) - \delta\Delta_1 PCM^{intertemporal}(t=2), & \text{when } t=1; \\ PCM^{myopic}(t=2) - \delta\Delta_2 PCM^{myopic}(t=3), & \text{when } t=2; \\ PCM^{myopic}(t=3), & \text{when } t=3. \end{cases} \end{aligned} \quad (26)$$

where

$$\Delta_t = \bar{\theta}_{r(t+1)rt} - \sum_{k=1, k \neq r}^J \bar{\theta}_{r(t+1)kt} \quad (27)$$

2. Generalizing to the T -period

The price-cost margin is defined as

$$p_t - mc_t = (p_t - w_t) + (w_t - mc_t)$$

$$= \begin{cases} PCM^{myopic}(t=1) - \delta\Delta_1 PCM^{intertemporal}(t=2), & \text{when } t=1; \\ PCM^{myopic}(t=2) - \delta\Delta_2 PCM^{intertemporal}(t=3), & \text{when } t=2; \\ \vdots & \vdots \\ \vdots & \vdots \\ PCM^{myopic}(t=T-1) - \delta\Delta_{T-1} PCM^{myopic}(t=T), & \text{when } t=T-1; \\ PCM^{myopic}(t=T), & \text{when } t=T. \end{cases} \quad (28)$$

where

$$\Delta_t = \bar{\theta}_{r(t+1)rt} - \sum_{k=1, k \neq r}^J \bar{\theta}_{r(t+1)kt} \quad (29)$$

2. Model Estimation in Ketchup Category

To show generalizability of the results, we performed the analysis for the ketchup category. This section provides details.

1. Data Description

We use A.C. Nielsen household panel and store level data in the Sioux Falls market over 85 weeks during the period of 1986-1987. Ketchup category has a relatively smaller number of UPCs and alleviate the possible bias when aggregating over different UPCs. We choose 4 largest UPCs (accounting for more than 90% of the market shares) in our study, while aggregating another 3 smaller UPCs into a composite brand. We restrict our analysis on the household purchase and store sales in the single largest store. Our household level data contain information on all shopping trips, including trips that did not lead to category purchase. In the case of category purchase, household level data has information on the UPC bought, price paid, feature and display that the household was exposed to, and coupon usage. Our store sales data contain information on the price, feature and display, and units sale data of these 5 UPCs in this store.

Table 2-1 describes household purchase data we used in the estimation. For household data, we select those households who made at least one category purchase in the sample period. This results in a sample of 1,074 households. These households made a total of 33,193 trips, including 2,798 trips (8.5% of all trips) which households made purchases in the ketchup category. We further calculate households' consumption rate per week based on their consumption over the sample period and then impute their inventory based on their consumption rate as well as purchases.

Table 2-1 gives the sample shares from household panel purchase data and aggregate shares from store sales data. Table 2-1 also gives the mean and standard deviations of the marketing mix, i.e. price, feature, and display, for the 5 UPCs in our data. As expected, Heinz 28 OZ has the highest unit price. In the 32OZ Ketchup sub-category, Heinz has the highest unit price, followed by Hunts and Del Monte.

Because ketchup products are quite similar in product attributes, in estimating the state-dependence model, we estimate state-dependence model using last purchase brand (same or different from current purchase), instead of the attribute-based similarity terms (as in the cereals category), in our estimation.

2. Results

Table 2-2 presents the demand model estimation results with endogeneity correction. Endogeneity correction is done using LIML method and we use the price data from another market (Springfield IL) as instrument variables for the prices in Sioux Falls. Hausman test rejects price exogeneity. Most of the parameter estimates have the expected signs.

Segments 2 and 3 (accounting for 96%) of the households are found to be inertial, while segment 1 (accounting for 4%) is found to be variety-seeking. This is consistent with most of the previous studies in packed-goods categories, which have found only inertia in purchase data (while in our study of the cereals category where we allow for a large number of choice alternatives, we find strong evidence of variety-seeking). After accounting for state-dependence, we find that consumers are less price sensitive than they appear in models without state-dependence as seen in the cereals category and in the simulations. Consumers in Segments 1 and 2 appear to be highly price insensitive, while consumers in Segment 3 are quite price sensitive.

Table 2-3 presents the supply model fit comparison results. We find that without accounting for state dependence, tacit collusion fits better than Bertrand pricing, whereas after accounting for state dependence, Bertrand pricing is clearly the best fitting game. This is directionally similar to the result in the cereal category in that not accounting for state dependence biases the results towards interpreting the behavior as tacit collusion. Further, just as in the cereal category, the one period look ahead model performed better than the myopic model.

Table 2-1: Summary Statistics of the Ketchup Purchase Data

		Heinz 32OZ	Hunts 32OZ	Heinz 28OZ	Del Monte 32OZ	Composite Brand	Outside Good		
Aggregate Shares	Mean	0.0314	0.0096	0.0146	0.0067	0.0186	0.9190		
	Stdev	0.0460	0.0063	0.0130	0.0093	0.0049	0.0441		
Sample Shares	Mean	0.0370	0.0097	0.0146	0.0070	0.0097	0.9245		
	Stdev	0.0595	0.0088	0.0184	0.0118	0.0058	0.0578		
							Weekly Variation	Brand Variation	
Price	Mean	0.0340	0.0328	0.0444	0.0306	0.0495	0.0380	0.0380	
	Stdev	0.0023	0.0037	0.0045	0.0032	0.0018	0.0025	0.0080	
Feature	Mean	0.1294	0.0588	0.0488	0.1176	0.0000	0.0706	0.0706	
	Stdev	0.3376	0.2367	0.2167	0.3241	0.0000	0.1010	0.0533	
Display	Mean	0.0353	0.0000	0.0122	0.0118	0.0000	0.0118	0.0118	
	Stdev	0.1856	0.0000	0.1104	0.1085	0.0000	0.0473	0.0144	
Consumption Rate* (units)	Mean	Across 1074 Households Over 80 weeks							
	Stdev	0.081431							
Inventory* (units)	Mean	0.0792753							
	Stdev	0.1176578							
		0.3032062							

Source: AC Nielsen

* Across 1074 Households over 80 weeks

Table 2-2: Demand-side Estimation

Parameter Estimates for Three-support Demand Model with Endogeneity Correction

	Variables	Three-segment with State-dependence			Three-segment without State-dependence		
		Estimate	std dev	t-statistics	Estimate	std dev	t-statistics
Segment 1	Heinz 32OZ	-9.4285	1.0279	-9.17	-5.4634	0.6694	-8.16
	Hunts 32OZ	-8.0958	0.8836	-9.16	-7.0248	0.7397	-9.50
	Heinz 28OZ	-8.3236	1.1007	-7.56	-7.6724	0.8141	-9.42
	Del Monte 32OZ	-5.3981	0.5676	-9.51	-6.4767	0.6632	-9.77
	Composite Brand	-3.8378	0.3822	-10.04	-7.4237	0.8535	-8.70
	price	-0.0399	0.7457	-0.05	-5.7434	4.1300	-1.39
	feature	0.4287	0.6329	0.68	1.6684	0.3952	4.22
	display	-0.0009	0.7567	0.00	-0.0451	1.0532	-0.04
	inventory	0.0877	0.3479	0.25	4.0971	0.8384	4.89
	intercept	-0.8445	1.4896	-0.57			
	family size	-0.1041	0.0946	-1.10			
	income	-0.8743	0.3301	-2.65			
	age	0.0918	0.0307	2.99			
children	6.1994	1.2571	4.93				
Segment 2	Heinz 32OZ	-4.9509	0.3814	-12.98	-6.6694	0.7914	-8.43
	Hunts 32OZ	-7.4136	0.5913	-12.54	-10.6090	1.3466	-7.88
	Heinz 28OZ	-6.8115	0.5917	-11.51	-7.3822	0.9029	-8.18
	Del Monte 32OZ	-7.6548	0.4858	-15.76	-9.9534	1.2227	-8.14
	Composite Brand	-7.6061	0.8465	-8.99	-5.4668	0.6641	-8.23
	price	-7.9270	5.3529	-1.48	-14.5530	12.1150	-1.20
	feature	2.8882	0.2595	11.13	5.2128	0.6857	7.60
	display	-0.3144	0.3596	-0.87	0.1022	0.3802	0.27
	inventory	-0.7573	0.1530	-4.95	-0.1945	0.1991	-0.98
	intercept	1.1714	0.9952	1.18			
	family size	-0.0796	0.0765	-1.04			
	income	0.3368	0.2023	1.66			
	age	-0.0287	0.0254	-1.13			
children	-0.4339	0.8815	-0.49				
Segment 3	Heinz 32OZ	-0.4649	0.1768	-2.63	-0.0776	0.1520	-0.51
	Hunts 32OZ	-1.6699	0.1609	-10.38	-1.3835	0.1577	-8.77
	Heinz 28OZ	-0.0188	0.1903	-0.10	0.1573	0.1705	0.92
	Del Monte 32OZ	-2.6310	0.1707	-15.42	-2.2665	0.1660	-13.65
	Composite Brand	-0.1476	0.2334	-0.63	0.2793	0.2316	1.21
	price	-86.7650	4.1275	-21.02	-89.6480	4.3819	-20.46
	feature	1.7296	0.0824	20.99	1.4077	0.1251	11.25
	display	0.6924	0.1546	4.48	0.5733	0.2032	2.82
	inventory	-1.5466	0.1138	-13.59	-1.4541	0.1305	-11.14
	intercept	1.8948	0.2526	7.50			
	family size	0.0038	0.0194	0.20			
	income	0.2208	0.0585	3.78			
	age	-0.0303	0.0052	-5.79			
children	-0.6302	0.1865	-3.38				
LL	logweight1	-3.2238	0.1650	-19.53	-3.1437	0.2144	-14.66
	logweight2	-0.6995	0.0691	-10.12	-0.7776	0.1342	-5.79
	size (segment 1)	0.0398			0.0431		
	size (segment 2)	0.4969			0.4595		
No. of Observations	33193						
				-9258.69			
						-9418.71	

Table 2-3
Supply Side Estimation: Fit Results

Manufacturer Interaction	Retailer Objective Manufacturer-Retailer Interaction	Log-Likelihoods (Vuong Test Statistics)		
		One-Period No State-Dependence	One-Period With State-Dependence	Two-Period With State-Dependence
Tacit Collusion	Category Profit Maximization	1251.7	1306.86	1371.84
	Manufacturer Stackelberg	(-)	(2.13*)	(1.96*)
Bertrand Competition	Category Profit Maximization	1228.5	1329.46	1390.70
	Manufacturer Stackelberg	(2.02*)	(--)	(--)

3. Continuous Heterogeneity

To address the concern as to whether the finite mixture model specification adequately accommodates heterogeneity and enables us to infer the right market elasticities, we compare the estimates of the discrete and continuous heterogeneity models. Whether continuous heterogeneity or discrete heterogeneity should be used in a particular analysis has been a topic of much debate and there are tradeoffs involved in using both types of models in the literature. Typical continuous heterogeneity models do not take multimodality of preference heterogeneity into account, while discrete heterogeneity models do not take within-segment heterogeneity into account.

In our application involving 117 brands, the computational complexity involved in estimating a continuous heterogeneity model is prohibitive and therefore practically impossible because it requires multiple weeks to estimate one demand model. In addition to doing simulation at each observation while using LIML, continuous random coefficient estimation requires also simulation at each household (i.e., if we make 30 draws, it's equivalent to estimating a 30-support latent class model). In order to correct endogeneity, at each observation, we need to make 30 draws to simulate the distribution of ξ . This requires 30×30 draws for each observation.

However to assure ourselves that this does not have any serious impact on our supply-side results, we sought to compare the elasticities from a discrete heterogeneity model with a continuous heterogeneity model. The mean price coefficient for the continuous heterogeneity model (-27.27) is comparable to the weighted mean price coefficient for the three-segment model (-24.08). The mean state-dependence parameter estimate (3.18) is close to the average state-dependence parameter estimate (3.49) from the 3-support estimates. The calculated price elasticities are also close to those from the three-support latent class (see Table 3-1). These results suggest that our three-support latent-class specification is sufficient for disentangling state-dependence and heterogeneity effects in the demand model.

Thus, similar to simulation studies with conjoint profiles (Andrews, Ansari and Currim, JMR 2002) and with scanner panel choice data (Andrews, Ainslie and Currim, JMR 2002), that compare the elasticities of the models with continuous and discrete heterogeneity, we find that the estimated elasticities are comparable. Andrews, Ainslie and Currim (2002, p. 487), note: "... We find that HB (*Hierarchical Bayes with Continuous Heterogeneity*) or FM (*finite mixture*) models have equally good parameter recovery and predictive validity, though the HB model has an advantage in fit. The findings also show that all model representations are quite robust to violations of underlying assumptions ... Whether an analyst prefers to use models with continuous or discrete representations of heterogeneity is a matter of opinion or personal

preference.” (italics ours).

Here are some illustrative elasticity estimates from continuous and discrete heterogeneity models for a few brands.

Table 3-1

Elasticity Estimates from Continuous and Discrete Heterogeneous Models

Elasticity Estimates	3-support Heterogeneous Specification of the Proposed Demand Model	Continuous Heterogeneous Specification of the Proposed Demand Model
General Mills Cheerios	-1.91	-2.41
General Mills Honey Nut Cheerios	-5.88	-5.49
General Mills Total	-5.71	-6.63
Kellogg’s Corn Flakes	-6.31	-5.97
Kellogg’s Raisin Bran	-6.39	-4.13
Kellogg’s Rice Krispies	-7.03	-6.15
Kellogg’s Frosted Flakes	-4.17	-4.59
Kellogg’s Special K	-7.48	-6.08
Quaker Life	-4.81	-4.31

4. Simulation: Demand Estimator Validation

The following simulation is to assess whether our estimator can recover the demand parameters. By keeping the simulated data to be similar in structure to the empirical data, we are able to assess whether the estimator can indeed recover the right model parameters for our data.

I. Data Generation Process

Assuming a sample of 600 households purchasing over 120 weeks over 117 brands and a no-purchase alternative, we simulate the households' choices, as well as the prices of 117 brands, after assuming the true values of the parameters to be identical to the parameter estimates obtained in our estimation.

The data are simulated as follows:

1. The lagged choices of the 600 households in period 1 are drawn from a multinomial distribution whose parameters represent average market shares of the 117 brands over the study period.
2. We assume arbitrary values for the prices of 117 brands over 120 weeks.
3. Period by period, we simulate each household's brand choice conditional on the period's price (obtained from the previous step), the household's product inventory, the segment-specific parameters of the household's demand function (depending on whichever of the 3 segments the household belongs to), and the household's lagged choice. In this manner, we simulate the 600 households' choices over 120 weeks.
4. Given the choice data from the previous step, the Bertrand equilibrium prices are computed for each period.
5. We check to see whether the prices computed in Step 4 are close to those used in Step 3 (within a pre-specified tolerance level). If they do, we save the simulated data. If not, we go back to Step 3, replace the existing prices with those from Step 4, and re-simulate households' brand choices. We keep cycling between Steps 3 and 4 until the prices between the two steps converge (within the pre-specified tolerance level).

To the extent that the purpose of this simulation is to check whether the estimator is able to correctly recover the true parameter values, we assume firms to be myopic (i.e., not forward-looking firms) in this simulation. However, we additionally allow for differences in households' relative brand-preferences across brands, degree of state-dependence in households' brand choices, and degree of forward-looking in firm behavior in two separate simulation exercises (reported in Appendices 5 and 6).

II. Results

The parameter estimates enclose the true values within the 95% confidence intervals (see Table 4-3).

Table 4-1

**Simulation with No Unobserved Characteristics
(True Values are assumed to be identical to our estimates)**

	Variables	True Value	State-dependence ($\xi=0$)			
			Estimates	std	lower bound	upper bound
Segment 1	Intercept	-6.85	-5.88	0.51	-6.91	-4.85
	sweetened	-0.12	-0.37	0.34	-1.04	0.31
	fiber	0.28	0.17	0.18	-0.19	0.53
	fruit/nut	0.24	0.38	0.35	-0.33	1.08
	price	-3.5	-3.05	0.27	-3.58	-2.51
Segment 2	Intercept	2.31	2.24	0.16	1.91	2.57
	sweetened	-0.33	-0.37	0.03	-0.44	-0.31
	fiber	0.08	0.11	0.03	0.05	0.17
	fruit/nut	0.02	-0.01	0.03	-0.07	0.06
	price	-1.37	-1.39	0.01	-1.42	-1.37
Segment 3	Intercept	9.02	6.96	0.67	5.62	8.31
	sweetened	1.22	1.56	1.78	-1.99	5.11
	fiber	-3.08	0.00	2.24	-4.47	4.47
	fruit/nut	0.61	0.96	0.33	0.31	1.62
	price	-85.46	-80.50	14.17	-108.84	-52.16
State Dependence	sd (support 1)	-1.46	-1.36	1.23	-3.82	1.10
	sd (support 2)	4.83	4.86	0.03	4.81	4.92
	sd (support 3)	0.68	1.49	1.73	-1.97	4.96
Size	log(weight1)	-1.2	-1.25	0.02	-1.28	-1.21
	log(weight2)	-1.8	-1.82	0.03	-1.87	-1.77
LL			-108962			

Numbers in bold signify that the true value lies within the 95% confidence interval of the estimate

Table 4-2

**Simulation with Unobserved Characteristics, No Endogeneity Correction
(True Values are assumed to be identical to our estimates)**

	Variables	True Value	State-dependence ($\xi \neq 0$, no endogeneity correction)			
			Estimates	std	lower bound	upper bound
Segment 1	Intercept	-6.85	-7.88	0.88	-9.63	-6.13
	sweetened	-0.12	0.02	0.40	-0.79	0.83
	fiber	0.28	0.16	0.35	-0.55	0.87
	fruit/nut	0.24	-0.05	0.27	-0.59	0.49
	price	-3.5	-4.03	0.37	-4.78	-3.28
Segment 2	Intercept	2.31	1.30	0.50	0.31	2.29
	sweetened	-0.33	-0.26	0.03	-0.32	-0.19
	fiber	0.08	0.12	0.03	0.06	0.18
	fruit/nut	0.02	-0.01	0.03	-0.07	0.06
	price	-1.37	-1.38	0.01	-1.41	-1.36
Segment 3	Intercept	9.02	3.97	1.55	0.86	7.08
	sweetened	1.22	4.04	1.59	0.86	7.22
	fiber	-3.08	0.00	2.24	-4.47	4.47
	fruit/nut	0.61	0.00	2.24	-4.47	4.47
	price	-85.46	-70.76	7.15	-85.06	-56.45
State Dependence	sd (support 1)	-1.46	-1.53	0.29	-2.11	-0.96
	sd (support 2)	4.83	4.88	0.03	4.82	4.94
	sd (support 3)	0.68	1.61	1.40	-1.20	4.41
Size	log(weight1)	-1.2	-1.20	0.01	-1.23	-1.17
	log(weight2)	-1.8	-1.97	0.03	-2.02	-1.91
LL			-68833.3			

Numbers in bold signify that the true value lies within the 95% confidence interval of the estimate

Table 4-3

**Simulation with Unobserved Characteristics and Endogeneity Correction
(True Values are assumed to be identical to our estimates)**

	Variables	True Value	State-dependence ($\xi \neq 0$, LIML method)			
			Estimates	std	lower bound	upper bound
Segment 1	Intercept	-6.85	-8.24	0.88	-9.99	-6.48
	sweetened	-0.12	-0.03	0.10	-0.24	0.17
	fiber	0.28	0.17	0.15	-0.13	0.47
	fruit/nut	0.24	-0.05	0.27	-0.59	0.49
	price	-3.5	-4.24	0.37	-4.99	-3.49
Segment 2	Intercept	2.31	1.24	0.65	-0.05	2.53
	sweetened	-0.33	-0.26	0.03	-0.33	-0.20
	fiber	0.08	0.12	0.03	0.06	0.18
	fruit/nut	0.02	-0.01	0.03	-0.07	0.06
	price	-1.37	-1.39	0.01	-1.41	-1.36
Segment 3	Intercept	9.02	4.33	1.65	1.02	7.64
	sweetened	1.22	4.20	1.51	1.17	7.22
	fiber	-3.08	-0.05	2.21	-4.47	4.38
	fruit/nut	0.61	-0.04	2.24	-4.51	4.43
	price	-85.46	-75.71	7.12	-89.95	-61.46
State Dependence	sd (support 1)	-1.46	-1.61	0.28	-2.17	-1.06
	sd (support 2)	4.83	4.89	0.04	4.82	4.97
	sd (support 3)	0.68	1.62	1.41	-1.21	4.44
Size	log(weight1)	-1.2	-1.24	0.02	-1.28	-1.21
	log(weight2)	-1.8	-1.89	0.03	-1.95	-1.84
LL			-46268.9			

Numbers in bold signify that the true value lies within the 95% confidence interval of the estimate

5. Simulation: Robustness of Supply Side Findings

We now perform a simulation to check whether the key supply side empirical results are robust to a wide range of parameter values.

Since it is difficult (and, to the best of our knowledge of the current literature, is not yet technically feasible) to solve the full dynamic programming decision problems of multiple manufacturers (in our case, more than 6), who also interact with a strategic retailer, we set up the simulation with only two brands and abstract away from strategic retailer behavior. Specifically, we assume a product category with two brands, each sold by a different manufacturer. We abstract away from the issue of strategic pricing by the retailer, and assume, instead, that the retailer charges a constant markup.

I. Data Generation:

Assuming a sample of 600 households purchasing over 120 weeks, we simulate data on households' brand choices and brands' prices under the following conditions:

1. Demand
 - a. Symmetric Brand intercepts: $\alpha_1 = \alpha_2 = -1$, Price coefficient: $\beta_p = -2$;
 - b. Asymmetric Brand intercepts: $\alpha_1 = -1, \alpha_2 = -2$, Price coefficient: $\beta_p = -2$
2. State Dependence: $sd = -1, -0.5, 0, 0.5$, and 1 allows for different levels of variety seeking, inertia and zero order behavior.
3. Firm Pricing: One period Forward Looking, Two periods Forward Looking

Under each of the $2*5*2 = 20$ treatment conditions, period by period, we solve for the equilibrium Bertrand prices of the two brands in the period (step 1), and then generate brand choices in that period for all households conditional on the solved prices and the assumed parameters (step 2). For the one-period forward-looking firm, the one-period Bertrand equilibrium that must characterize week 2 must be computed for each possible state of market shares that the two firms could find themselves in at the end of week 1.¹ In this manner, the value function of each firm at each possible state in each period is computed by backward induction, which leads to the optimal price vector of each firm over the 2 weeks. Prices generated in this manner can be thought of as a Markov-perfect equilibrium of a 2-period repeated game in prices between firms. The same backward induction procedure is used to generate prices over 100 weeks in the simulation data. Overall, we solve 99 dynamic games (starting in weeks 1, 2, 3, ... etc.). In other words, we solve for open-loop (and not closed-loop)

¹ Since market shares are continuous, we discretize them over 100 uniformly spaced grid-points over the range $[0, 1]$. This leads to 10,000 possible states that firms can find themselves in at the end of each period in the dynamic program.

pricing policies of firms. We then simulate households' brand choices conditional on the open-loop equilibrium prices of firms over all the weeks.

II. Estimation:

Demand Model:

Given the simulated choice data, we estimate the demand model with and without state dependence. Since there are 20 simulated datasets, this leads us to estimate 40 different demand models. The estimated demand models with state dependence are able to correctly recover the demand functions used to simulate the choice data. On the other hand, the estimated demand models without state dependence are unable to recover the true demand functions (except under the condition of $sd = 0$), thus highlighting the biases that arise in the estimates of mis-specified demand models. Specifically, the estimate of the price coefficient always turns out to be larger in magnitude than the true value, which is consistent with previous findings in the state dependence literature (see, for example, Seetharaman, Ainslie and Chintagunta 1999). For the sake of readability, we do not report the estimation results for the demand model here. These are available from the authors upon request.

Supply Model:

Given the demand estimates with and without state dependence, we estimate the supply model under alternative assumptions of firm behavior, as shown below.

1. Firms' Forward Looking Behavior and Accounting for State Dependence

- a. Static firms ignoring state dependence (Static w/o SD)
- b. Myopic firms taking state dependence into account (Myopic w/ SD),
- c. Dynamic firms (one period or two period as appropriate; we do not report one period estimates when the actual data was based on two periods and vice versa) (Dynamic)

2. Type of Pricing Interaction

- a. Bertrand
- b. Collusive

Therefore, there are $3*2 = 6$ possible pricing games between firms. Given that there are 20 treatment conditions for the simulated data, and we have 6 pricing games under each condition, this leads us to estimate $20*6 = 120$ pricing games. The maximized log-likelihood values for these 120 estimated pricing games are reported in Tables 5-1-5-4.

III. Results:

We now present the simulation results (Tables 5-1-5-4). The two key results are as follows:

1. Collusive games fit best when state dependence is ignored (Static w/o SD), but Bertrand games fit best when state dependence is accounted for (Myopic w/ SD,

Dynamic). This is true for both inertia and variety seeking, and for low (0.5, -0.5) versus high (1, -1) levels of state dependence.

2. The average improvement in log-likelihood obtained from accounting for state dependence (Static w/o SD to Myopic w/ SD) is much greater than that obtained from accounting for forward-looking behavior (Myopic w/ SD to Dynamic). This result is summarized below.

	Static w/o SD to Myopic w/ SD	Myopic w/ SD to Dynamic
Symmetric Demand – One Period Look Ahead	9.1%	1%
Symmetric Demand – Two Period look Ahead	32.5%	2.5%
Asymmetric Demand – One Period look Ahead	43.2%	2.1%
Asymmetric Demand – Two Period look Ahead	32%	2.2%

Table 5-1

Symmetric brands, One-period forward-looking firms

	$\alpha_1 = -1, \alpha_2 = -1, \beta_p = -2$	Static Pricing Model Without SD	Myopic Pricing Model With SD	Dynamic Pricing Model
SD=1	Bertrand	360.3795	399.7086	410.0263
	Collusive	399.8537	383.9778	401.9989
SD=0.5	Bertrand	398.2264	420.0372	420.0372
	Collusive	402.3753	402.3753	391.7333
SD=0	Bertrand	406.9743	406.9743	417.0377
	Collusive	397.9653	397.9653	408.2255
SD=-0.5	Bertrand	382.2101	408.4432	412.9365
	Collusive	408.4814	401.4218	406.9569
SD=-1	Bertrand	363.1280	409.4105	411.5427
	Collusive	410.0754	401.4503	405.0152

Table 5-2

Symmetric brands, Two-period forward-looking firms

	$\alpha_1 = -1, \alpha_2 = -1, \beta_P = -2$	Static Pricing Model Without SD	Myopic Pricing Model With SD	Dynamic Pricing Model
SD=1	Bertrand	243.3238	373.7465	388.1266
	Collusive	372.2712	335.6801	332.1062
SD=0.5	Bertrand	323.3734	358.1879	372.4616
	Collusive	358.7591	303.4585	322.5452
SD=0	Bertrand	407.6288	407.6288	417.6193
	Collusive	353.4808	353.4808	333.8565
SD=-0.5	Bertrand	265.3185	408.1028	414.8450
	Collusive	354.0206	343.3753	350.3795
SD=-1	Bertrand	365.0493	408.1156	409.4209
	Collusive	401.0860	399.2521	400.57364

Table 5-3**Asymmetric brands, One-period forward-looking firms**

	$\alpha_1 = -1, \alpha_2 = -2, \beta_p = -2$	Static Pricing Model Without SD	Myopic Pricing Model With SD	Dynamic Pricing Model
SD=1	Bertrand	287.1233	410.0227	421.8320
	Collusive	365.5848	364.5221	366.3707
SD=0.5	Bertrand	297.2908	407.5921	410.6581
	Collusive	359.4930	362.5847	363.6016
SD=0	Bertrand	406.1670	406.1670	412.5198
	Collusive	351.4604	351.4604	360.3133
SD=-0.5	Bertrand	306.0747	407.4201	415.3221
	Collusive	396.3834	362.8361	363.9184
SD=-1	Bertrand	255.6781	408.1990	418.7097
	Collusive	359.5510	393.8348	363.1561

Table 5-4

Asymmetric brands, Two-period forward-looking firms

	$\alpha_1 = -1, \alpha_2 = -2, \beta_p = -2$	Static Pricing Model Without SD	Myopic Pricing Model With SD	Dynamic Pricing Model
SD=1	Bertrand	197.6899	289.8209	295.2024
	Collusive	259.2422	181.8640	197.5464
SD=0.5	Bertrand	233.5827	334.1050	346.0724
	Collusive	284.6311	282.9182	280.2167
SD=0	Bertrand	407.8188	407.8188	410.1475
	Collusive	354.2216	354.2216	362.8317
SD=-0.5	Bertrand	320.5818	398.0804	408.2747
	Collusive	346.9247	348.3671	262.7864
SD=-1	Bertrand	357.3427	408.7998	412.1222
	Collusive	359.3497	358.3181	357.3065

6. Simulation: Sensitivity to Alternative Forward Looking Horizons of Manufacturers

Our empirical analysis sequentially tested 1 period, 2 period, 3 period forward-looking models. We did not find much improvement in fit by moving beyond the 2 period look-ahead models. However one question is whether forward looking behavior in the actual data may have had a much greater periodicity: For example, it may be possible that a manufacturer might set wholesale prices for the entire quarter by looking a quarter ahead. We simulate such a scenario and find that indeed if the data are generated based on a one quarter look ahead, the estimator will detect such behavior, suggesting that our results are not due to a different time horizon used by firms.

For the purpose of this simulation exercise, we assume a product category with two brands, each sold by a different manufacturer. We abstract away from the issue of strategic pricing by the retailer, and assume, instead, that the retailer charges a constant markup.

I. Data Generation:

Assuming a sample of 600 households purchasing over 120 weeks, we simulate data on households' brand choices and brands' prices under the following conditions:

1. Demand
 - a. Symmetric Brand intercepts: $\alpha_1 = \alpha_2 = -1$, Price coefficient: $\beta_p = -2$;
 - b. Asymmetric Brand intercepts: $\alpha_1 = -1, \alpha_2 = -2$, Price coefficient: $\beta_p = -2$
2. State Dependence: $sd = -1, -0.5, 0, 0.5, \text{ and } 1$ allows for different levels of variety seeking, inertia and zero order behavior.
3. Firm Pricing: Manufacturers set prices for weeks 13-24 in week 1, for weeks 25-36 in week 13 and so on.

Under each of the $2*5*1 = 10$ treatment conditions, we solve for each brand's weekly prices, quarter by quarter, starting from the quarter representing weeks 13-24, and then solving the firm's dynamic program by backward induction within the quarter. For this, the one-period Bertrand equilibrium that must characterize week 12 must be computed for each possible state of market shares that the two firms could find themselves in at the end of week 11.² In this manner, the value function of each firm at each possible state in each period is computed by backward induction, which leads to the optimal price vector of each firm over the 12 weeks of the quarter. Prices generated in this manner can be thought of as a Markov-perfect equilibrium of a 12-period repeated game in prices between firms. The same backward induction procedure is used to generate prices over all 10 quarters in the simulation data. In other words, since

promotion calendars involve the idea of firms pre-committing (to the retailer) their wholesale prices over a quarter, we solve 10 mutually exclusive dynamic games (starting in weeks 13, 25, 37 etc.), instead of having to solve 108 dynamic games (starting in weeks 13, 14, 15 etc.). In other words, we solve for open-loop (and not closed-loop) pricing policies of firms. This is consistent given the institutional concern that firms may make price decisions at the beginning of each quarter for the entire quarter ahead. We then simulate households' brand choices conditional on the open-loop equilibrium prices of firms over all the weeks in the quarter.

II. Estimation:

Demand Model:

Given the simulated choice data, we estimate the demand model with and without state dependence. Since there are 10 simulated datasets, this leads us to estimate 20 different demand models. The estimated demand models with state dependence are able to correctly recover the demand functions used to simulate the choice data. On the other hand, the estimated demand models without state dependence are unable to recover the true demand functions (except under the condition of $sd = 0$), thus highlighting the biases that arise in the estimates of mis-specified demand models. Specifically, the estimate of the price coefficient always turns out to be larger in magnitude than the true value, which is consistent with previous findings in the state dependence literature (see, for example, Seetharaman, Ainslie and Chintagunta 1999). For the sake of readability, we do not report the estimation results for the demand model here. These are available from the authors upon request.

Supply Model:

Given the demand estimates with and without state dependence, we estimate the supply model under alternative assumptions of firm behavior, as shown below.

1. Firms' Forward Looking Behavior and Accounting for State Dependence

- a. Static firms ignoring state dependence (Static w/o SD)
- b. Myopic firms taking state dependence into account (Myopic w/ SD),
- c. Dynamic firms with one period look ahead (Dynamic w/ 1-Week)
- d. Dynamic firms with two period look ahead (Dynamic w/ 1-Quarter)

2. Type of Pricing Interaction

- a. Bertrand
- b. Collusive

² Since market shares are continuous, we discretize them over 100 uniformly spaced grid-points over the range $[0, 1]$. This leads to 10,000 possible states that firms can find themselves in at the end of each period in the dynamic program.

Therefore, there are $4 \times 2 = 8$ possible pricing games between firms. Given that there are 10 treatment conditions for the simulated data, and we have 8 pricing games under each condition, this leads us to estimate $10 \times 8 = 80$ pricing games. The maximized log-likelihood values for these 80 estimated pricing games are reported in Tables 6-1-6-2.

III. Results:

We now present the simulation results (Tables 6-1-6-2). The two key results are:

1. Collusive games fit best when state dependence is ignored (Static w/o SD), but Bertrand games fit best when state dependence is accounted for (Myopic w/ SD, Dynamic). This is true for both inertia and variety seeking, and for low (0.5, -0.5) versus high (1, -1) levels of state dependence.
2. The average improvement in log-likelihood obtained from accounting for state dependence (Static w/o SD to Myopic w/ SD) is much greater than that obtained from either (a) accounting for forward-looking behavior (Myopic w/ SD to Dynamic w/ 1-week), or (b) allowing for a longer time horizon of forward planning (Dynamic w/ 1-week to Dynamic w/ 1-Quarter). The first result, i.e., (a), mirrors the finding in Appendix B. The second result, i.e., (b), says that even when firms forward plan for an entire quarter's worth of their brands' prices, the one-week look ahead model appears to be a good approximation (i.e., provides a reasonable model fit) of such a pricing process. One reason for this, as argued in the main text of our paper, could be that the strategic effects due to oligopolistic competition may substantially offset the effects on a brand's prices due to a firm's forward-looking behavior with other firms.³

	Static to Myopic w/ SD	Myopic w/ SD to Dynamic w/ 1 Wk	Dynamic w/ 1 Week to Dynamic w/ 1 Qtr
Symmetric Demand – One Quarter Look Ahead	28%	6%	3%
Asymmetric Demand – One Quarter Look Ahead	31.7%	9.2%	4.7%

³ As we show in the empirical analysis, the cost estimates do not change much between forward-looking games and myopic pricing games because strategic effects dominate forward-looking behavior due to state-dependence. When there are no strategic effects, forward-looking pricing due to state-dependence will cause cost estimates to change a lot, as shown in Berry and Pakes 2000.

Table 6-1**Symmetric Brands, 1-Quarter Look Ahead**

	$\alpha_1 = -1, \alpha_2 = -1, \beta_p = -2$	Static Pricing Model Without SD	Myopic Pricing Model With SD	Dynamic Pricing Model 1-week Look-Ahead	Dynamic Pricing Model 12-week (1-quarter) Look-Ahead
SD=1	Bertrand	212.7631	222.5728	232.0337	237.5745
	Collusive	221.5343	132.3927	171.3478	175.2059
SD=0.5	Bertrand	216.7031	264.9713	267.2561	276.8153
	Collusive	233.0042	219.5643	252.0929	255.5102
SD=0	Bertrand	392.6351	392.6351	393.2543	394.9351
	Collusive	382.0480	382.0480	382.7587	389.9887
SD=-0.5	Bertrand	255.4928	372.2830	377.4574	388.6194
	Collusive	276.2034	354.5784	239.9269	233.8873
SD=-1	Bertrand	257.5070	359.2209	380.8095	390.5228
	Collusive	267.5476	306.3249	326.6660	345.0881

Table 6-2**Asymmetric Brands, 1-Quarter Look Ahead**

	$\alpha_1 = -1, \alpha_2 = -2, \beta_P = -2$	Static Pricing Model Without SD	Myopic Pricing Model With SD	Dynamic Pricing Model 1-week Look-Ahead	Dynamic Pricing Model 12-week (1-quarter) Look-Ahead
SD=1	Bertrand	146.8939	168.8902	175.4306	195.4770
	Collusive	158.8731	162.7988	165.7161	161.3895
SD=0.5	Bertrand	106.2944	198.0499	210.6272	214.4837
	Collusive	135.7274	161.2529	166.3449	156.7879
SD=0	Bertrand	237.9310	237.9310	238.7903	239.1443
	Collusive	215.8793	215.8793	197.7238	211.6744
SD=-0.5	Bertrand	208.8075	213.1070	224.8201	235.2079
	Collusive	217.2195	205.9454	196.8513	217.2842
SD=-1	Bertrand	311.1474	383.1917	391.4128	396.1538
	Collusive	323.5052	340.0369	344.0856	390.4197