Supply Chain Collaboration Under Information Asymmetry

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Abstract

Firms manufacturing highly innovative and complex products often rely on the expertise of their suppliers who provide critical components that enable core functionality of products. During the product design stage there is often considerable uncertainty about component production cost, and it is of interest to both the manufacturer and the supplier to enter into a collaborative relationship in order to reduce this uncertainty and the expected cost. Despite the benefit, however, the supplier might be fearful of revealing his proprietary cost information and thus may be reluctant to fully collaborate. Building on the traditional frameworks of the newsvendor model and adverse selection, we investigate how information asymmetry and contracting strategies affect collaboration outcomes. We compare contracts based on price and quantity, and find that the manufacturer may be better off committing to a price early (before the conclusion of product development) since this promotes full collaboration. However, delaying the contract offer can also be a viable strategy if (a) collaboration leads to a large decrease of expected production cost but not of cost uncertainty, and (b) large demand uncertainty exists. We also find that, paradoxically, efforts to reduce demand uncertainty and production lead time may hinder collaboration during product development.

1 Introduction

Manufacturing firms engage in diverse working relationships with suppliers and their strategic market positions may critically depend on how these relationships are managed. For instance, it has been well documented that the subcontracting structure of Japanese automobile manufacturers, in which suppliers actively participate in every development and production process, has been one of the key differentiators enabling their competitive advantage over U.S. manufacturers (McMillan 1990). With a gradual shift toward outsourcing business models and increasingly complex components required by customized products, the traditional market-driven, competition-based approach to procurement is being reevaluated.

Collaboration, which transcends this traditional approach, is one of the terms most frequently mentioned in the press and among the practitioners of strategic sourcing. Although the term is used to describe a wide range of relationships between a buyer (whom we refer to as a manufacturer in this paper) and a supplier, it usually means that firms do not maintain purely transactional activities. Co-branding efforts, joint research programs, and long-term strategic alliances are among the manifestations of collaboration (Rudzki 2004). Given that approximately 80% of a product's lifecycle cost is determined early in the product design stage (Blanchard 1978), it is no surprise that firms often focus on collaboration during product development and product lifecycle planning. Dyer (2000), for one, provides evidence that early product development collaboration decreases the number of late-stage engineering changes, hence contributing to cost reduction. Thus, collaboration is especially important for firms manufacturing products such as industrial equipment, automobiles, aircraft, and consumer electronics, which are highly innovative and complex in nature and therefore require coordinated efforts among supply chain members to reduce cost. As a matter of fact, a recent survey (Aberdeen Group 2006) shows that managers view cost reduction as one of the primary goals that they hope to achieve through collaboration.

Depending on the characteristics of the market they belong to, the degree to which a manufacturer and a supplier collaborate during product development may vary. At one extreme, firms hold each other at "arm's length" and they communicate about little other than product specification. In such an environment, firms rely on contractual safeguards to enforce agreements (Dyer et al. 1998). At the other end of the spectrum, firms find it to their mutual advantage to fully collaborate, sharing their expertise and finding ways to improve the quality and cost of a product together. They are motivated to do so because the success of each firm is intertwined with the other. In the words of an operations director from Copeland Corporation, an Ohio-based manufacturer of AC/heating equipment, on collaborating with their supplier Osco Foundries (Kinni 1996, p. 105):

> The only way we could reach the current state of manufacturing efficiency is through sharing and understanding both companies' processes... Osco is a member of the New Product Team and is intimately involved in all aspects of the casting design and machining process. The best way to achieve the lowest-cost raw material and finished component is to leverage the design process by utilizing the supplier's expertise and achieving the lowest true cost for the component.

While firms strive to attain successful collaboration, it can be an elusive goal. Success usually rests on two key factors: incentives and characteristics of business environment. The first refers to the degree to which firms are willing to enter into and sustain the relationship. The benefits of collaboration notwithstanding, the reality that the ultimate goal of each firm is to maximize its own profit means that firms are averse to sharing all information and are opportunistic. At the same time, collaboration partners may be tempted to take advantage of their private knowledge, such as cost information, since it is impossible for one party to monitor all the data and activities of the other. As an executive from an auto parts supplier put it, "if one doesn't say anything, all the savings are ours" (Anderson and Jap 2005). From these reports and other evidence, it is clear that information asymmetry plays a critical role in determining firms' incentives to collaborate.

In addition, environmental characteristics such as the strategic importance of a procured component, the modularity of component architecture, and uncertainty in production cost, quality, and delivery lead time also play a big role in determining successful outcome of collaboration (Pyke and Johnson 2003). In this paper we focus on the impact of uncertainty, both of demand and of the cost of producing a strategically important component. Demand uncertainty is especially important for innovative products with short lifecycles such as smartphones, whose fast pace of feature evolutions and unpredictable consumer tastes create difficulty in demand forecasting and inventory risks due to high rates of obsolescence. Uncertainty in component cost, which is incurred by the supplier, arises as the supplier faces a multitude of production options early in the collaborative process, especially when the product is initially designed. For instance, the supplier may consider acquiring or developing untested new technology in order to fulfill the manufacturer's expectation for the end product's functionality. In fact, uncertainty arising from new technology adoption is cited as one of the main drivers of collaboration among supply chain members (Handfield et al. 1999, Ragatz et al. 2002).

The factors we have mentioned – uncertainty, information asymmetry, and incentives to collaborate – are all intricately related. As the supplier's initial uncertainty about production cost stems in large part from imprecise product design requirements provided by the manufacturer,[1] cost uncertainty is likely to be reduced by closer interactions with the manufacturer which would help with clearing up ambiguity and selecting suitable technology. Additionally, such an increase in predictability is typically accompanied by reduction of expected unit cost, as evidenced by many studies including Handfield et al. (1999), which reports in their survey of 49 manufacturers that collaboration reduced production costs by up to 30%. Reduction of cost, both in average and in uncertainty, is perhaps the most important benefit of collaboration during the product development stage. Despite this advantage, however, collaboration may not always be a winning proposition to the supplier. As the collaborative process involves mutual information exchanges, the supplier unavoidably reveals some of his cost structure to the manufacturer. For example, the supplier may have to inform the manufacturer that he will use a particular material to build a component, but the price of the material may be known publicly. This puts the supplier in a precarious situation since he may be heavy-handed by the manufacturer later during procurement contract negotiation; having a better estimate of the supplier's cost, the manufacturer may fully utilize that knowledge to structure contract terms that will boost her own profit and leave a smaller share of the supply chain profit to the supplier. Fearing such an outcome, the supplier may guard against collaborating too closely with the manufacturer. Therefore, the supplier faces a dilemma: despite obvious advantages, should he actually collaborate, and if he does, to what extent? What contracting strategy can the manufacturer employ to convince the supplier to collaborate? How do uncertainties in demand and cost impact collaboration decision? These are the questions that we investigate in this paper.

[1] During the product development stage, suppliers usually receive only rough estimates of design specification parameters from the OEM's (Nellore et al. 1999).

In this paper we propose a stylized model that formalizes the idea of the degree of collaboration. Using this model, we find that the supplier's incentive to collaborate critically depends on the manufacturer's contracting strategy. We consider two key approaches: the precommitment strategy and the delay strategy. With the former, the manufacturer commits to either one or both of component price and order quantity early, during the product design stage. With the latter, the manufacturer waits until design is complete to offer a procurement contract. We first show that the delay strategy is always optimal if the supply chain is integrated. In a decentralized setting, however, the delay strategy may or may not be superior to precommitment, especially price precommitment. This is because the delay strategy engenders a high degree of opportunistic behaviors by both parties; although the delay strategy has the potential to bring more profit to the manufacturer, it also causes the supplier to be more reluctant to collaborate. On the other hand, price precommitment eliminates such adversarial motivations, but the manufacturer is not able to gain as much supply chain profit as she could have. Hence, there are pros and cons to these two contracting strategies. In fact, no one strategy dominates the other in all circumstances. We find that the delay strategy is optimal for the manufacturer only when (a) collaboration brings significant reduction of expected unit cost but not of cost uncertainty, and (b) large demand uncertainty exists. Otherwise, she prefers price precommitment. Extending this insight further, we also find that, paradoxically, efforts to reduce demand uncertainty and production lead time may hinder collaboration if the delay strategy is employed. These results highlight the role of incentives in a decentralized supply chain; by contrast, the delay strategy is found to be always optimal in a centralized setting. Therefore, we find that one of the well-established principles in the operations management literature – postponing decisions until uncertainty is reduced or removed – does not always apply when supply chain members act in self-interested manners.

The rest of the paper is organized as follows. In Section 2 we survey the related literature. In Section 3 we lay out model assumptions and introduce notations used throughout the paper. After establishing the results of the benchmark case in Section 4, we present the analysis of the main model in Section 5. Section 6 provides further insights based on the findings in Section 5. Finally, we conclude the paper in Section 7 discussing managerial insights and opportunities for further research.

2 Literature Review

Observations that firms join forces to achieve a common goal have received much attention in both the business press and the academic literature. Reflecting its popularity, this concept goes by different names: strategic alliance, partnership, joint venture, and collaboration, to name just a few. Many papers touch upon similar ideas, encompassing distinct areas. For example, a large stream of literature has analyzed Collaborative Planning, Forecasting and Replenishment strategies (CPFR, see Aviv 2007). While our work is motivated by the similar broad concept, we focus on a specific issue that arises in the context of product development and procurement contracting. Namely, we aim to understand how firms' incentives to collaborate in the early product design stage are influenced by uncertainties in cost and demand as well as information asymmetry. Our focus is motivated by the tremendous importance and widespread implementation of supplier cost reduction initiatives in practice (see, for example, Stallkamp 2005 for a detailed description of the supplier cost reduction effort program SCORE at Chrysler).

We focus on collaborative efforts that are made during the product design stage that precedes the production stage. In this respect, our paper is related to the new product development (NPD) literature. Surprisingly, the topic of inter-firm collaboration has not received much attention in the NPD literature (for surveys of the literature, see Krishnan and Ulrich 2001 and Krishnan and Loch 2005). The only exception, to the best of our knowledge, is Bhaskaran and Krishnan (2009), whose broad theme is similar to ours but who investigate a set of research questions quite different from ours. While they acknowledge that agency issues caused by the opportunistic behavior of development partners are a real challenge in collaboration, they sidestep this issue. In contrast, information asymmetry plays a central role in our paper. In addition, procurement contracting (i.e., price and quantity decisions) is absent in their model, whereas it is an integral feature in ours.

Supply chain contracting in the presence of information asymmetry has become an established area of research in OM in recent years. Our model fits most closely with adverse selection models. Articles such as Ha (2000), Corbett (2001), and Corbett et al. (2004) represent early works in this stream of research. Among them, Iyer et al. (2005) is quite related to this paper since they also consider the use of a screening contract in the context of product development. However, their focus is on the impact of complementarity/substitutability between buyer resources and supplier capability rather than on collaboration. More recently, several authors have investigated multi-period dynamic adverse selection problems arising in strategic sourcing, just as we do in this paper. Li and Debo (2009) compare sole- and second-sourcing strategies when long-term contracting is viable. Taylor and Plambeck (2007a,b) go beyond formal contracting, studying optimal relational contracts in repeated, infinite-horizon settings. Although there are some similarities (for example, Taylor and Plambeck 2007a compare price-only and price-quantity contracts, as do we), these models differ from ours in many dimensions, including motivations, modeling approaches, and managerial insights.

One of the central elements of our model is the contract offer timing decision, which naturally brings up the hold-up problem (Klein et al. 1978) and the issue of evaluating operational flexibility vs. the value of precommitment. In the operations literature, Taylor (2006) examines this issue in a setting where a manufacturer may contract with a retailer either before or after demand is realized. He finds conditions under which the manufacturer prefers to sell late (i.e., offer a contract late) given that the retailer possesses private information about demand and either does or does not exert a sales effort. Although we also discuss how environmental factors influence the contract offer timing decision, the similarity ends there since our results are driven by completely different dynamics, e.g., interaction between demand and cost uncertainties. The work that comes closest to ours in addressing the timing issue is Gilbert and Cvsa (2002). Our paper differs from theirs in many respects, however, especially in our focus on information asymmetry, the role of uncertainty originating not only from demand but also from cost, and the mismatch between demand and supply captured by the newsvendor framework. Another related paper is Wang at al. (2009), which considers the manufacturer's involvement into supplier reliability improvement and also allows for early and late commitment into production quantities. Likewise, this paper does not allow for information asymmetry and focuses on uncertainty due to supplier reliability rather than costs.

3 Model Assumptions

Our modeling timeline consists of two stages: the product design stage and the production stage. A manufacturer ("she") designs and builds a highly innovative product. Once the product is built at the end of the production stage, it is sold at price r, which is exogenously determined. Its demand

D is a random variable with the cdf F and the pdf f with $\overline{F}(\cdot) \equiv 1 - F(\cdot)$. This distribution is common knowledge. Let $\mu \equiv E[D]$. We assume that F has a support $[0, \infty)$ with $F(0) = 0$, and that it exhibits an increasing generalized failure rate (IGFR) property, an assumption which is satisfied by many well-known distributions. The manufacturer assembles q units of the product before the selling season. At the end of the selling season, unsold units are lost. The product is composed of multiple parts, one of which is a key component that has the unit cost c (elaborated below). The manufacturer also incurs the cost of acquiring the remaining parts and the cost of assembling the product, but we normalize these to zero since they do not play significant roles in our analysis.

3.1 Collaboration Level and Unit Production Cost

Because the manufacturer lacks in-house expertise or resources to develop and manufacture the key component of the product in order to introduce the product to the market in time, she outsources the task to a supplier ("he"), who possesses the necessary capability. We assume that each end-product requires one unit of this component. The manufacturer communicates the functional requirement for the component to the supplier. In the initial stage of the collaboration process the supplier does not have sufficient knowledge about how to manufacture the component in the most efficient manner due to the innovative and complex nature of the product. For example, the supplier may have a number of options in terms of which materials to use to build the component, how to design the architecture of the component so that it integrates well into the rest of the assembled product, or which subcontractor to use for the procurement of needed parts. As a result, the unit production cost c of producing a component is highly uncertain at the onset of the design collaboration process. However, this uncertainty can be reduced by improving the level of collaboration, i.e., by continually exchanging ideas and learning each other's expectations and limitations. Moreover, a higher level of collaboration usually brings the additional benefit of reducing the expected per unit production cost. Therefore, collaboration leads to reduction of both the mean and the variance of the unit cost. In our model, we assume that such changes in the unit cost function are the main outcomes of collaboration and specifically focus on them.

To formalize this idea, we introduce a parameter $\theta \in [0, 1]$ that denotes the level of collaboration between the manufacturer and the supplier. At $\theta = 0$ the firms are completely disengaged (the arm's length relationship), whereas $\theta = 1$ corresponds to full collaboration. Although increasing the level of collaboration typically incurs extra costs (e.g., costs associated with personnel exchanges, communication, travel, etc.), we normalize these to zero since they do not affect qualitative insights.[2] In the product design stage in which the collaboration occurs, the unit cost c is uncertain and is a function of θ. In particular, $c \,|\, \theta$ is a random variable with the conditional cdf $G(\cdot \,|\, \theta)$ and the pdf $g(\cdot \,|\, \theta)$. We assume that the conditional reverse hazard rate $g(\cdot \,|\, \theta)/G(\cdot \,|\, \theta)$ is decreasing, a condition that is satisfied by many well-known distributions. At each θ, $c \,|\, \theta$ has a finite support $[\underline{c}_\theta, \overline{c}_\theta]$. Note that we can specify any realization of $c \,|\, \theta$ once the quantile of the distribution is known, as the mapping $G^{-1}(z \,|\, \theta)$ uniquely identifies the unit cost realization at the z^{th} quantile and at θ, for $z \in [0, 1]$. Throughout the paper we present our model in the transformed (θ, z)-space instead of the original (θ, c)-space because doing so simplifies analysis. With this transformation, the lower and upper bounds on $c \,|\, \theta$ are $\underline{c}_\theta = G^{-1}(0 \,|\, \theta)$ and $\overline{c}_\theta = G^{-1}(1 \,|\, \theta)$. As a function of θ, $G^{-1}(\cdot \,|\, \theta)$ also represents the *equiquantile curve* in the original (θ, c)-space, i.e., the collection of unit cost realizations having the same quantile along the θ-axis. For brevity, we refer to $G^{-1}(\cdot \,|\, \theta)$ as the unit cost function.

The following properties of $G(\cdot \,|\, \theta)$ capture that collaboration leads to reductions in both the mean and the variance of the unit cost.

Assumption 1 *(i)* $\frac{\partial}{\partial \theta} E[c \,|\, \theta] = \frac{\partial}{\partial \theta} \int_0^1 G^{-1}(z \,|\, \theta) dz < 0$ *and (ii)* $\frac{\partial}{\partial \theta} [G^{-1}(z_2 \,|\, \theta) - G^{-1}(z_1 \,|\, \theta)] < 0$ *for* $0 \leq z_1 < z_2 \leq 1$.

Property (ii) says that the gap between any two equiquantile curves, $G^{-1}(z_2 \,|\, \theta) - G^{-1}(z_1 \,|\, \theta)$, which we call a *spread* at θ, becomes narrower with higher θ. In other words, $G(\cdot \,|\, \theta)$ exhibits decreasing dispersive order in θ (Müller and Stoyan 2002, p. 40). By this definition, reduction of the unit cost variance is equivalent to reduction of the spread.[3]

While these two conditions are sufficient to generate all the qualitative results demonstrated in this paper, the general functional form of G, combined with the general functional form of F, does not lend itself to tractable analytical expressions in the game-theoretic model that we set

[2] As a result, $\theta = 1$ or the term "full collaboration", which occurs throughout the paper, should be understood in the context of the zero collaboration cost assumption.

[3] That $G(\cdot \,|\, \theta_2)$ is smaller than $G(\cdot \,|\, \theta_1)$ in dispersive order implies that the mean-adjusted unit cost $c \,|\, \theta - E[c \,|\, \theta]$ has a smaller variance for $\theta_2 > \theta_1$ (Müller and Stoyan 2002, Theorem 1.7.6 and Corollary 1.5.4).

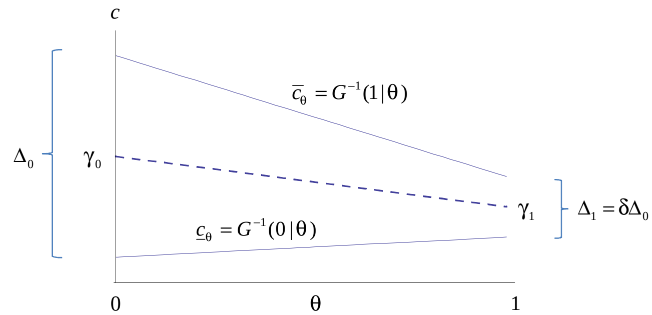


Figure 1: An example of a possible unit cost function satisfying Assumption 2.

up below. For this reason, we make more restrictive assumptions for the remainder of the paper. These assumptions together imply the conditions in Assumption 1.

Assumption 2 *(i) $G(\cdot\,|\,\theta)$ has a uniform distribution with a mean $\gamma_\theta \equiv E[c\,|\,\theta]$ and support $[\underline{c}_\theta, \bar{c}_\theta]$ for all $\theta \in [0,1]$.*

(ii) $G^{-1}(z\,|\,\theta)$ is linear in θ for all $z \in [0,1]$.

(iii) $\gamma_0 > \gamma_1$.

(iv) $\delta \equiv \Delta_1/\Delta_0 < 1$, where $\Delta_\theta \equiv \bar{c}_\theta - \underline{c}_\theta$.

See Figure 1 for an illustration. Uniform distribution in (i) is frequently employed for analytical tractability in the models in which two or more random variables interact, as in ours (see, for example, Li and Debo 2008, Cachon and Swinney 2009). Under this assumption, the expected unit cost at a given θ is at $z = 1/2$, i.e., $\gamma_\theta = G^{-1}(1/2\,|\,\theta)$. With (ii), we assume that all equiquantile curves are linear in the (θ, c)-space. This assumption is not too restrictive because only the relative values of θ are of interest; replacing them with nonlinear monotonic equiquantile curves only changes the scale. For example, a family of convex or concave equiquantile curves that can be expressed in a common functional form (such as exponential) can be transformed into a family of linear curves with an appropriate rescaling of the θ-coordinate. With (i) and (ii), the assumptions (iii) and (iv) are sufficient conditions for Assumption 1 to hold, i.e., they ensure that the mean and the spread of the uniformly distributed unit cost $c\,|\,\theta$ are reduced with higher θ. Note that Δ_1 in (iv) represents the residual uncertainty left at $\theta = 1$ and δ is the ratio between this residual uncertainty and the uncertainty at $\theta = 0$. With $\delta < 1$, part (ii) of Assumption 1 is satisfied. Under these assumptions, we can express $G^{-1}(z \,|\, \theta)$ as

$$G^{-1}(z \,|\, \theta) = \gamma_0 - (\gamma_0 - \gamma_1)\theta + \Delta_0 \left(1 - (1 - \delta)\theta\right)(z - 1/2). \tag{1}$$

The term $\gamma_0 - (\gamma_0 - \gamma_1)\theta$ is the expected unit cost at θ. The last term $\Delta_0 \left(1 - (1 - \delta)\theta\right)(z - 1/2)$ is the spread between the mean and the z^{th} quantile of the unit cost at θ, representing the unit cost uncertainty. This linearization allows for analytically tractable analysis. Finally, we assume that $\gamma_0 > \Delta_0/2$, $\gamma_1 > \Delta_1/2$, and $\gamma_0 + (3/2)\Delta_0 < r$, to ensure that unit cost and order quantity are always positive.

We assume that the manufacturer and the supplier have a common knowledge regarding how the distribution $G(\cdot \,|\, \theta)$ changes with θ in the product design stage, possibly because they agree on a cost improvement projection at the beginning. We also assume that the actual change in $G(\cdot \,|\, \theta)$ realized after collaboration does not deviate from the initial projection. Throughout the product design stage, the two parties have access to the same information about the unit cost, namely, its distribution. Information asymmetry emerges at the end of that stage, after collaboration is completed and before the supplier begins production of the component. At that point, the supplier privately learns the realized cost, or equivalently, the quantile z given θ. Hence, the realized $z \,|\, \theta$ represents the supplier's "type". This information is not relayed to the manufacturer, and the supplier keeps it to himself with the intention of using it to his advantage. The manufacturer continues to have only limited knowledge about the supplier's type, i.e., the distribution $G(\cdot \,|\, \theta)$.

3.2 Contracting

The manufacturer, as the principal who has a larger stake in the success of the product and has bigger market presence, determines the terms of the procurement contract. However, it is the supplier who decides whether and how much to collaborate, since he is the one providing the necessary capability to build the product and has an upper hand in controlling how much proprietary knowledge to share.[4]

We assume that the collaboration level θ is not contractible because it is not directly observed

[4] With its SCORE program in the 1990's, Chrysler encouraged its suppliers to voluntarily define ideas for contributing to cost reduction (Stallkamp 2005).

by the manufacturer. This is a plausible assumption since firms typically cannot monitor their counterpart's internal decisions. However, the manufacturer can indirectly infer the supplier's choice of θ by guessing the corresponding bounds of the unit cost, i.e., $\underline{c}_\theta$ and $\bar{c}_\theta$, based on information obtained from her interactions with the supplier. We assume that the manufacturer's guess is consistent with the supplier's actual choice. This is quite likely since the supplier has little incentive to mislead the manufacturer during collaboration; by openly discussing available options for technologies and resources to meet the manufacturer's requirements, the supplier can in return receive cost-saving information tailored for each option. The manufacturer can then estimate the costs associated with the proposed technologies/resources from public sources or by other means. These assumptions also rule out the use of a contract based on the estimated cost bounds, since the manufacturer does not obtain the bounds directly from the supplier (e.g., the manufacturer does not examine the supplier's accounting books or the innovation process) and cannot verify them.

Motivated by the majority of procurement practices, we assume that the contract type is that of the price-quantity pair, (w, q). That is, the manufacturer specifies the unit price and the quantity of the procured component, making sure that the supplier will agree to the proposed contract terms. Depending on when the manufacturer offers the contract, the contract may or may not consist of a single price-quantity pair. If she delays the contract offer to the beginning of the production stage, at which point the supplier has chosen θ and information asymmetry about the unit cost is in place, the best contract for the manufacturer has a screening form, i.e., a menu of price-quantity pairs $\{(w(z \mid \theta), q(z \mid \theta))\}_{0 \leq (z,\theta) \leq 1}$, each pair mapping to the supplier's type $z \mid \theta$. As is well known in the mechanism design literature (Bolton and Dewatripont 2005), the manufacturer can structure the menu so that the supplier chooses the pair designed for him, thereby truthfully revealing his type, and the manufacturer extracts all of the supplier's surplus except for his *information rent*, which represents the inefficiency created by information asymmetry.

Although offering a procurement contract at the beginning of the production stage is routinely observed in practice, it is not the only option available to the manufacturer. In particular, she may decide to offer a contract early in the relationship, i.e., at the beginning of the product design stage. She may also decide to offer two contract terms at separate times, one early in the product design stage and the other in the production stage. We use the term *precommitment strategies* to refer to the collection of strategies under which at least one of the contract terms is

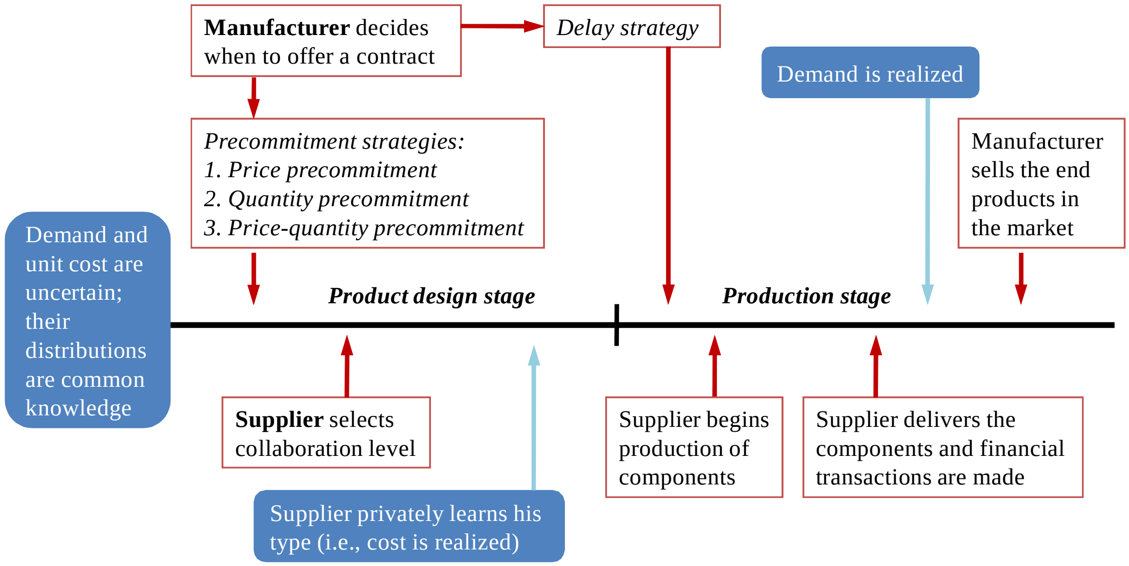


Figure 2: Sequence of events.

offered in the beginning. Therefore, there are three such strategies: price precommitment, quantity precommitment, and price-quantity precommitment. Variants of these strategies are observed in practice. For instance, Japanese auto manufacturers and their suppliers agree on a payment amount based on the projected cost improvement that they expect to achieve through joint efforts (Womack et al. 1991). A similar practice was adopted by Chrysler as part of its pre-sourcing effort (Dyer 2000). Volume commitments are also frequently used as a way to improve supply chain relationships (Corbett et al. 1999). As we will see later, these precommitment strategies as well as the delay strategy each creates different incentives to collaborate.

To summarize, the manufacturer decides which contracting strategy to adopt before commencing product design collaboration with the supplier. She can either precommit price, quantity, or both, or offer a contract specifying both price and quantity after collaboration is completed. Once the strategy is set, the manufacturer and the supplier collaborate during the product design stage, if the supplier decides to do so. At some point in the design stage the supplier learns the true cost, but he keeps this information from the manufacturer. In the production stage, the supplier manufactures and delivers the components in the quantity specified in the contract, and receives payment. The manufacturer in turn assembles the end products and sells them in the market. (See Figure 2 for a summary of the sequence of events.) We ask the following questions: How do different contracting strategies impact the supplier's incentive to collaborate? Given the supplier's incentive, which contracting strategy is best for the manufacturer, and under what circumstances? We investigate these questions in Sections 5, after analyzing the benchmark case.

4 Benchmark Case: Integrated Firm

We first establish a benchmark under the assumption that the manufacturer and the supplier constitute an integrated firm. It is convenient to regard the "supplier" as a subdivision within the firm that specializes in component design, communicating with a supervising division which, acting as a strategic planner for the firm, establishes the degree of collaboration and production quantity. While the optimal collaboration level is determined in the product design stage, there are two possibilities for the timing of the quantity decision, mirroring the contracting strategies described in Section 3.2: choose the quantity either in the product design stage before the unit cost is realized (the precommitment strategy), or in the production stage after the unit cost is realized (the delay strategy). With the first option, the integrated firm faces the problem

$$(\mathcal{IP}) \qquad \max_{\theta,q} \; rE[\min\{D,q\}] - \left(\int_0^1 G^{-1}(z\,|\,\theta)dz\right) q,$$

whereas with the second option, the problem is

$$\begin{aligned} (\mathcal{ID}) \qquad & \max_{\theta} \qquad && \int_0^1 \left(rE[\min\{D,q(z\,|\,\theta)\}] - G^{-1}(z\,|\,\theta)q(z\,|\,\theta)\right) dz \\ & \text{s.t.} && q(z\,|\,\theta) = \arg\max \left\{rE[\min\{D,q\}] - G^{-1}(z\,|\,\theta)q\right\}. \end{aligned}$$

Note that $(\mathcal{ID})$ can be viewed as a stochastic program with recourse (Birge and Louveaux 1997). Unfortunately, it is analytically intractable to obtain a complete analytical specification of the solution of $(\mathcal{ID})$ for all possible unit cost functions allowed under Assumption 2. However, essential insights can be gained from the two special cases whose analytical solutions are available.

Proposition 1 *(i) If $\gamma_0 - \gamma_1 \geq (1-\delta)\Delta_0/2$, the integrated firm chooses the delay strategy, setting $\theta^I = 1$ and choosing the quantity $q^I(z) = F^{-1}\left(1 - \frac{1}{r}\left[\gamma_1 + \delta\Delta_0\left(z - \frac{1}{2}\right)\right]\right)$.*

(ii) In the limit $\gamma_0 - \gamma_1 \to 0$, the integrated firm chooses the delay strategy, setting $\theta^I = 0$ and

choosing the quantity $q^I(z) = F^{-1}\left(1 - \frac{1}{r}\left[\gamma_0 + \Delta_0\left(z - \frac{1}{2}\right)\right]\right)$.

In each case, the expected firm profit at the optimum is $\Pi^I = r\int_0^1 \mu(q^I(z))dz$*, where* $\mu(y) \equiv \int_0^y xf(x)dx$ *is the incomplete mean of* D.

The condition in (i) implies that all equiquantile curves are nonincreasing, while the condition in (ii) means that the expected unit cost is unchanged with θ. The first situation arises when the rate of reduction of the expected unit cost is sufficiently higher than that of cost uncertainty. The second situation arises when only cost uncertainty is reduced with a higher θ. In between these two extreme instances there are cases in which the equiquantile curves above a certain threshold quantile decrease, while the curves below it increase.

According to the proposition, the delay strategy is preferred by the integrated firm in both cases. Numerical examples show that this conclusion holds for the cases not covered by the proposition as well. This conclusion is quite intuitive; given an opportunity to wait until cost uncertainty disappears, the firm chooses to do so and takes advantage of being able to react to the realized cost before determining the production quantity. Such a delay is a common feature of many well-known operational strategies, including the quick response in the fashion retailing industry (Fisher and Raman 1996, Iyer and Bergen 1997) and the delayed differentiation in the consumer electronics industry (Lee and Tang 1999).

The collaboration level choice in (i) is intuitive too. Since the unit cost realization is lowest at $\theta = 1$ for all quantiles in this case, the firm is likely to attain the highest expected profit at that level as the expected demand-supply mismatch cost is minimized.[5] The collaboration level choice in (ii), however, is not obvious. It states that, given that the expected unit cost does not change with θ, the firm prefers the collaboration level $\theta = 0$ where cost uncertainty is *highest*. This choice reflects the risk-seeking behavior of the firm which is inherent in the newsvendor model. To explain further, consider the standard newsvendor problem $\max_q \Pi(q \mid c) = rE[\min\{D, q\}] - cq$ (with an abuse of notation, c here represents the realized unit cost rather than a random variable). The solution is $q^* = F^{-1}(1 - c/r)$. By the envelope theorem, $\frac{\partial}{\partial c}\Pi(q^* \mid c) = \frac{\partial}{\partial c}\Pi(q \mid c)\big|_{q=q^*} = -q^*$. Then $\frac{\partial^2}{\partial c^2}\Pi(q^* \mid c) = -\frac{\partial}{\partial c}q^* = 1/rf(q^*) > 0$, implying that $\Pi(q^* \mid c)$ is convex in c. A convex utility function of an agent means that he prefers a lottery to certainty (Gollier 2001).

[5] With a lower unit cost, three things happen: (1) underage cost increases, (2) overage cost decreases, and (3) optimal quantity increases. Combined, the net effect is a smaller risk of lost sales and a lower cost of leftover inventory.

Therefore, *a newsvendor facing cost uncertainty before quantity decision is a risk-seeker*. Intuitively, a newsvendor is willing to bet on a low cost realization that would bring her the benefit of reduced demand-supply mismatch cost, knowing that she can minimize the possible damage caused by a high cost realization through adjusting the production quantity later.

Combining the results of (i) and (ii), we infer that the integrated firm with the unit cost function as specified in Assumption 2 weighs the benefit of lower expected unit cost against her propensity to take a risk when she chooses the optimal level of collaboration. The condition in (i) ensures that the former dominates the latter, while the opposite is true under the condition in (ii). Does a middle ground exist in between? Consistent with the convex utility function of a newsvendor, numerical examples show that the integrated firm's expected profit function has a convex shape, i.e., the optimal θ is always found at the end points. In addition, $\theta^I = 1$ is observed in the majority of cases where both the mean and the variance of the unit cost decrease with θ, a more realistic scenario in practice. Hence, we believe that the case (i), in which the integrated firm chooses the delay strategy and full collaboration, is a more appropriate benchmark in general. We return to the case (ii) in Section 5.3.

5 Analysis

Having gained insight about the benchmark case in the previous section, we now turn to the decentralized supply chain setting. In two successive subsections (Sections 5.1 and 5.2), we analyze the incentive effects under the different contracting strategies mentioned earlier. In Section 5.3, we compare the equilibrium outcomes of the strategies to determine which is preferred by the manufacturer.

5.1 Precommitment Strategies

We first consider the precommitment strategies, under which the manufacturer commits to a contract term early in the relationship before the supplier makes the collaboration decision. The early commitment may be on price, quantity, or both. The equivalent strategy in the benchmark case has already proved suboptimal (see Proposition 1); for an integrated firm, committing to a quantity well before the unit cost is realized is inferior since doing so carries a risk that can be avoided if the firm waits until uncertainty disappears. The same risk exists in the decentralized setting, so precommitment continues to be costly for the manufacturer as she abandons the option to wait. However, it is a priori unclear if the supplier subject to a particular contracting strategy is motivated to fully collaborate with the manufacturer, contrary to what the manufacturer may desire. If the supplier is not motivated to collaborate, such an incentive reduces the effectiveness of the strategy as it may prevent the manufacturer from reaching her full cost savings potential. Therefore, the key to discovering if the delay strategy continues to be optimal in the decentralized setting is to understand the supplier's motivation to collaborate.

First, consider price-quantity precommitment. Since no information asymmetry exists at the time the manufacturer offers the contract, the screening mechanism is unnecessary so the optimal contract consists of a single pair (w^p, q^p) that is independent of the supplier's type z, which is yet to be realized. The manufacturer's problem is

$$(\mathcal{P}) \qquad \max_{(w,q)} \quad rE[\min\{D, q\}] - wq$$

$$\text{s.t.} \quad \pi_s(z \,|\, \theta^p) \geq 0 \ , \ \forall z \in [0,1], \tag{IR-P}$$

$$\theta^p = \arg\max \ \left\{ \int_0^1 \pi_s(z \,|\, \theta) dz \right\} , \tag{IC-P}$$

where $\pi_s(z \,|\, \theta) = (w - G^{-1}(z \,|\, \theta))q$ is the z-type supplier's ex-post profit. The condition (IC-P) states that the supplier chooses the optimal θ in the product design stage to maximize his expected profit in response to the manufacturer's contract offer. The participation constraint (IR-P) ensures that the supplier makes at least what he can earn from an outside opportunity, which is normalized to zero, regardless of the realized cost. By imposing this constraint, we rule out a scenario in which the supplier refuses to participate in the trade after collaborating with the manufacturer.[6] This formulation is consistent with the optimization problem under the delay strategy (in the next subsection) where a similar ex-post participation constraint appears. It is straightforward to show that the manufacturer's expected profit increases in θ, i.e., she strictly prefers full collaboration. The reason is similar to that of the integrated firm case: under Assumption 3, full collaboration leads

[6] Adding the ex-ante participation constraint $\int_0^1 \pi_s(z \,|\, \theta^p)dz \geq 0$ in addition to (IR-P) is redundant, since it is implied by (IR-P). On the other hand, replacing (IR-P) with the ex-ante participation constraint is problematic since it opens up the possibility that the supplier will walk away from the trade if his realized cost is too high.

to the most cost savings, minimizing the expected demand-supply mismatch cost and generating the highest profit potential for the manufacturer.

The manufacturer's contract design problems under the price commitment and the quantity commitment strategies are similar to ($\mathcal{P}$) except that one more step is added to each problem to account for the fact that contract parameters are determined at separate times. (Note that the screening mechanism does not apply to these cases either since only one of the two contract terms is offered when information asymmetry exists, i.e., in the production stage.) We omit the formal problem definitions due to space limitation and go directly to the results. First, we summarize the supplier's optimal choice of collaboration level under different precommitment strategies.

Lemma 1 *Under all precommitment strategies, the manufacturer prefers $\theta = 1$. However, the supplier chooses $\theta = 0$ under quantity precommitment while he chooses $\theta = 1$ under the price precommitment and price-quantity precommitment strategies.*

Surprisingly, quantity precommitment and price precommitment have the opposite effects on the supplier's collaboration decision. Given that the manufacturer strictly prefers full collaboration, the lemma shows that the incentive of a supplier under price precommitment or price-quantity precommitment is aligned with the manufacturer's incentive, whereas incentives are not aligned under quantity precommitment. The underlying dynamics of these results are as follows. Consider quantity precommitment first. Since quantity is fixed at the time the supplier chooses the collaboration level, the only other factor influencing the supplier's decision is his profit margin. If the supplier increases the level of collaboration, unit cost uncertainty becomes smaller (by Assumption 2) as he exposes more information about his cost to the manufacturer. The manufacturer will then take advantage of this information and will subsequently lower the price, thus driving down the supplier's profit margin. Fearing such an outcome, the supplier abstains from collaborating in order to maximally guard his informational advantage, i.e., he chooses $\theta = 0$, whereby cost uncertainty is highest. This is a manifestation of the classical hold-up problem; as the supplier anticipates the manufacturer's opportunistic behavior, he has little incentive to collaborate even though he expects that cost savings could be achieved.

Now let us consider price precommitment. It turns out that committing to a price early also means committing to an order quantity because, once the price w is set, so is the optimal quantity, which is determined from the newsvendor fractile $(r-w)/r$. Therefore, price precommitment and price-quantity precommitment are equivalent in our setting. Given that all contract terms are effectively fixed in the beginning, the supplier then chooses the collaboration level that leads to the highest cost savings in expectation: $\theta = 1$. The hold-up problem does not exist in this case because price precommitment guarantees that the manufacturer will not behave opportunistically later. Therefore, Lemma 1 suggests that price precommitment and price-quantity precommitment are superior strategies for the manufacturer compared to quantity precommitment. The following proposition formalizes this result.

Proposition 2 *Among the precommitment strategies, price precommitment and price-quantity precommitment are optimal for the manufacturer. Under both strategies, the manufacturer offers* $w^p = \gamma_1 + \frac{\delta\Delta_0}{2}$ *and* $q^p = F^{-1}\left(1 - \frac{w^p}{r}\right)$*, while the supplier chooses* $\theta^p = 1$*. The resulting expected profit of the manufacturer is* $\pi_m^p = r\mu(q^p)$*.*

Therefore, price precommitment and price-quantity precommitment have the same equilibrium outcomes and they dominate quantity precommitment. For this reason, in what follows we will refer to price precommitment when we use the term "precommitment" except when ambiguity arises.

5.2 Delay Strategy

Let us turn our attention to the delay strategy. As shown in Section 4, this strategy is optimal for the integrated firm as the firm can take advantage of cost realization. Although the same benefit exists under the delay strategy, there is an important difference in the decentralized supply chain: the realized cost is known only to the supplier. As a result, the manufacturer may not be able to extract the maximum surplus via contracting. In this case, the optimal contract consists of a menu of price-quantity pairs $\{(w^*(z\,|\,\theta), q^*(z\,|\,\theta))\}$, which are determined from the following optimization problem:

$$(\mathcal{D}_m) \qquad \max_{\{(w(z\,|\,\theta),q(z\,|\,\theta))\}} \quad \int_0^1 \left(rE[\min\{D, q(z\,|\,\theta)\}] - w(z\,|\,\theta)q(z\,|\,\theta)\right) dz$$

$$\text{s.t.} \quad \pi_s(z,z\,|\,\theta) \geq 0\ ,\ \forall z \in [0,1], \tag{IR-D}$$

$$\pi_s(z,z\,|\,\theta) \geq \pi_s(z,\widehat{z}\,|\,\theta)\ ,\ \forall z, \widehat{z} \in [0,1]. \tag{IC-D}$$

Here, $\pi_s(z, \widehat{z} \,|\, \theta) = [w(\widehat{z} \,|\, \theta) - G^{-1}(z \,|\, \theta)]q(\widehat{z} \,|\, \theta)$ is the z-type supplier's profit (i.e., his information rent) when he accepts the price-quantity pair $(w(\widehat{z} \,|\, \theta), q(\widehat{z} \,|\, \theta))$ that is intended for the $\widehat{z}$-type supplier. Note that θ is already chosen by the supplier at the time the screening contract is offered. Imposing the constraint (IC-D) is a direct result of the revelation principle, which says that there is an optimal contract under which the supplier picks the pair designed for him, thereby revealing his true type. The participation constraint (IR-D) ensures that the resulting profit for the supplier is nonnegative regardless of the realized cost. The optimal contract is specified in the following lemma.

Lemma 2 *Let* $\nu(z \,|\, \theta) \equiv \Delta_0 (1 - (1-\delta)\theta)\, z$. *Given* θ, *the manufacturer offers the contract terms* $q^*(z \,|\, \theta) = F^{-1}\left(1 - \frac{1}{r}\left[G^{-1}(z \,|\, \theta) + \nu(z \,|\, \theta)\right]\right)$ *and* $w^*(z \,|\, \theta) = G^{-1}(z \,|\, \theta) + \frac{\nu(z \,|\, \theta)}{zq^*(z \,|\, \theta)} \int_z^1 q^*(x \,|\, \theta)dx$. *The resulting expected profits of the manufacturer and the supplier are* $\pi_m^*(\theta) = r\int_0^1 \mu(q^*(z \,|\, \theta))dz$ *and* $\pi_s^*(\theta) = \int_0^1 \nu(z \,|\, \theta)q^*(z \,|\, \theta)dz$.

Recall that the supplier's ex-post profit under the optimal screening contract consists entirely of his information rent. Hence, $\pi_s^*(\theta)$ represents the supplier's expected information rent for a given θ, as evaluated in the product design stage (before the unit cost is realized). In the product design stage, the supplier, anticipating that the manufacturer will offer the screening contract $\{(w^*(z \,|\, \theta), q^*(z \,|\, \theta))\}$ computed from $(\mathcal{D}_m)$, chooses the optimal collaboration level to maximize his expected profit. In other words, he solves

$$(\mathcal{D}_s) \qquad \max_{\theta} \int_0^1 [w^*(z \,|\, \theta) - G^{-1}(z \,|\, \theta)]q^*(z \,|\, \theta)dz.$$

Let θ^d be the solution of this problem. Unfortunately, a complete solution specification is difficult to obtain, as it was in the benchmark case. For this reason, we focus on a special case that admits a tractable analysis. The following additional assumption characterizes this case.

Assumption 3 *The lower bound of the unit cost is constant (does not vary with* θ*).*

Combined with Assumption 2, this is equivalent to assuming $\gamma_0 - \gamma_1 = (1-\delta)\Delta_0/2$ so that all equiquantile curves are nonincreasing in θ. Note also that this condition satisfies the condition in part (i) of Proposition 1, the benchmark against which the results should be compared. Let $\underline{c}$ be the value of the lower bound. Then $G^{-1}(z \,|\, \theta)$ in (1) is simplified to $G^{-1}(z \,|\, \theta) = \underline{c} + \Delta_0 \left(1 - (1-\delta)\theta\right) z$. As will become clear shortly, we are able to capture the most important aspects of the incentive dynamics with this choice of unit cost function. One serious limitation is that we cannot distinguish between the impact of the reduction of the expected unit cost and the impact of the reduction of cost uncertainty, as the two are tied under Assumption 3. Therefore, we offer qualified statements in this subsection whenever appropriate.

Proposition 3 *The manufacturer prefers $\theta = 1$ under the delay strategy. Suppose that Assumption 3 holds. Then there exists a unique $0 \le q^\dagger < F^{-1}\left(1 - \underline{c}/r\right)$ that solves*

$$J(q) \equiv q\left(\overline{F}(q) - \frac{\underline{c}}{r}\right)^2 - \int_q^{F^{-1}(1-\underline{c}/r)} \left(\overline{F}(x) - \frac{\underline{c}}{r}\right) x f(x) dx = 0. \tag{2}$$

If $q^\dagger \le F^{-1}\left(1 - \frac{1}{r}(\underline{c} + 2\Delta_0)\right)$, then the supplier who is offered the menu of contracts specified in Lemma 2 chooses $\theta^d = 0$. If $q^\dagger \ge F^{-1}\left(1 - \frac{1}{r}(\underline{c} + 2\delta\Delta_0)\right)$, then $\theta^d = 1$. Otherwise, $0 < \theta^d < 1$ and

$$\theta^d = \frac{\underline{c} + 2\Delta_0 - r\overline{F}(q^\dagger)}{2\Delta_0(1-\delta)}. \tag{3}$$

Let $\pi_m^d \equiv \pi_m^*(\theta^d)$ and $\pi_s^d \equiv \pi_s^*(\theta^d)$ be the expected profits of the manufacturer and the supplier at the equilibrium. The proposition states that the supplier may choose θ anywhere between 0 and 1, depending on parameter values. In fact, the supplier always chooses $\theta < 1$ if $\delta = 0$; full collaboration never occurs if the unit cost at $\theta = 1$ is perfectly known to be $\underline{c}$. This finding is in striking contrast with the conclusion from the benchmark case, which showed that full collaboration is optimal for the integrated firm when the delay strategy is used (see Proposition 1, part (i)). Again, this is an example of the hold-up problem caused by information asymmetry. What differentiates this result from those of the precommitment strategies is that the optimal collaboration level is not necessarily at the extremes (i.e., $\theta = 0$ or $\theta = 1$). On one hand, it is intuitive that a middle ground is reached under the delay strategy since we found earlier that opposite collaboration outcomes arise under price precommitment and quantity precommitment; by precommitting neither, a convex combination of the two extremes may obtain. However, this reasoning is too simplistic.

To explain further, suppose that the supplier decides to choose a high collaboration level θ. Under Assumptions 2 and 3, this action leads to a decrease in the unit cost realization $G^{-1}(z \,|\, \theta)$ of a given supplier type z as well as in the spread around z. As discussed in the previous subsection, reduction of the spread (hence of cost uncertainty) adversely affects the supplier since smaller uncertainty provides the opportunistic manufacturer with better ability to lower the price, leaving little profit margin to the supplier. In fact, this "squeezing" is very effective under the delay strategy (compared to quantity precommitment, under which a similar hold-up situation arises) since the manufacturer can utilize the screening mechanism, which leaves theoretically minimum surplus to the supplier and therefore forces the supplier to resist collaboration. However, this is not the entire story; there is a secondary effect that did not exist under the precommitment strategies, namely, a volume increase. Since lower unit cost realizations under higher θ prompt the manufacturer to lower contract prices, this in turn implies higher underage cost and lower overage cost for her. Therefore, the manufacturer increases the order quantity. This effect is new because quantity was essentially fixed under all precommitment strategies (see discussion below Lemma 1). Combining the two effects, we see that an increased collaboration level causes a trade-off between volume increase and margin reduction. Therefore, collaboration is a double-edged sword for the supplier employing the delay strategy; he has to trade off a potential profit increase resulting from a reduction of the expected unit cost with a profit decrease due to erosion in his informational advantage.

Next, let us investigate how the optimal collaboration level θ^d is impacted by changes in the values of important environmental variables. In particular, we focus on the two variables that represent the degree of uncertainty, namely, the fractional residual cost uncertainty δ and the standard deviation of the demand distribution.

Proposition 4 *For $\theta^d \in (0,1)$, (i) $\partial\theta^d/\partial\delta > 0$ and (ii) $\partial\theta^d/\partial\sigma > 0$ when $D \sim$Normal(μ,σ).*

Note that, for convenience, we only consider what happens to the interior solution of θ^d in this proposition. In (ii), we restrict our attention to the normal demand distribution because it allows us to obtain the result analytically. We have numerically verified that non-normal distributions do not alter this result. Consider (i) first. While smaller δ means smaller cost uncertainty at any fixed θ, under Assumption 3, it also implies lower expected unit cost, since all equiquantile curves move downward with smaller δ. Therefore, a decrease in δ creates the same effect as does an increase in θ. However, the supplier actually reacts in the opposite direction and distances himself from the manufacturer (as he chooses lower θ with smaller δ, according to (i)), which strongly suggests that his prime concern is protecting his informational advantage, which is reduced with lower δ. In fact, this is consistent with our earlier observation that full collaboration never occurs when $\delta = 0$. We will return to this point in the next subsection to make a more concrete statement.

According to part (ii), the supplier decides to collaborate more closely with the manufacturer as demand becomes more uncertain. On the surface, it sounds quite intuitive that larger demand uncertainty leads to closer collaboration. However, this result is not at all driven by usual reasons such as demand information sharing (for example, Lee et al. 2004 identifies the collaborative effort to share demand forecasts as one of the strategies for mitigating the bullwhip effect) since there is no consideration of demand information sharing in our model. Instead, the underlying mechanism for this result involves a subtle interaction between demand uncertainty and variations of the unit cost as a function of θ under Assumptions 2 and 3. In brief, higher demand uncertainty causes higher demand-supply mismatch risk for the manufacturer, prompting her to compensate for the loss by finding a way to lower the unit cost (see footnote 5). This is done by restructuring the terms of the screening contract so that they are more appealing to a supplier who collaborates more closely, as higher collaboration increases the likelihood of realizing lower unit cost. A more detailed reasoning is provided in Appendix A.

Together, (i) and (ii) in Proposition 4 suggest that the conditions conducive to a high level of collaboration under the delay strategy consist of (a) large residual cost uncertainty, and/or (b) large demand uncertainty. These insights will prove to be valuable in the next subsection.

5.3 The Optimal Contracting Strategy

Having studied the incentive effects of each contracting strategy, we are now in a position to determine which strategy is preferred by the manufacturer. Because of analytical complexity, we present the discussion using a mix of formal analysis and numerical examples. We begin with two boundary cases that sharpen the intuition: $\delta \to 1$ with $\gamma_0 > \gamma_1$ and $\gamma_0 \to \gamma_1$. The former case is a situation in which collaboration brings reduced expected unit cost but no changes in cost uncertainty. The latter is the other extreme: the only benefit of collaboration is reduced cost uncertainty. As it turns out, we can obtain analytical solution only for the former case.

Proposition 5 *If $\gamma_0 > \gamma_1$ and $\delta \to 1$, the manufacturer chooses the delay strategy and the supplier* *chooses* $\theta = 1$.

The proposition makes it clear that it is reduction of cost uncertainty that holds back the supplier from collaborating under the delay strategy. In an extreme situation where cost uncertainty is unaffected by collaboration, as considered in the proposition, the supplier finds it optimal to fully collaborate *regardless of demand uncertainty*. This happens because the supplier's information advantage remains the same while the order volume is expected to increase (higher θ corresponds to lower cost, hence higher underage cost and lower overage cost for the manufacturer). Since it is guaranteed that full collaboration will occur, demand uncertainty does not enter into the manufacturer's contract design consideration. In turn, the manufacturer, expecting that the supplier will fully collaborate under both the precommitment and delay strategies, prefers the latter strategy since she can utilize the screening mechanism to efficiently extract the supplier's surplus.

Now consider $\gamma_0 \to \gamma_1$, i.e., when collaboration results in the reduction of unit cost uncertainty but not of expected unit cost. From numerical experiments, we find in this case that the supplier chooses not to collaborate under the delay strategy, i.e., $\theta^d = 0$, regardless of demand uncertainty. Notice that this result is the opposite of what we found in the other extreme case, $\gamma_0 > \gamma_1$ with $\delta \to 1$. This outcome is consistent with our findings thus far; in the absence of the benefit of a lower expected unit cost, the supplier chooses a low collaboration level to protect his private information best, which is at $\theta = 0$. Also consistent is that, as verified from numerical experiments, the manufacturer actually desires the opposite of what the supplier prefers, i.e., $\theta = 1$. It is interesting to compare this result with that of the benchmark case. Recall from part (ii) of Proposition 1 that $\theta = 0$ was the outcome under the same circumstance if the supply chain was integrated because a newsvendor firm exhibits risk-seeking behavior toward cost uncertainty. Therefore, the fact that the manufacturer's expected profit increases in θ in a decentralized supply chain indicates that her risk-seeking nature is suppressed when she deals with a supplier who has private information. To be more precise, the benefit of having better visibility into the supplier's cost structure (which happens at $\theta = 1$) dominates the manufacturer's propensity to take risk. Summarizing the outcome, we come to the following somewhat counterintuitive conclusion. If the only benefit of collaboration is lower cost uncertainty, under the delay strategy, the supplier chooses the collaboration level that the manufacturer does not want (i.e., $\theta = 0$) but that would have been preferred by her if the supplier chain were integrated instead. Of course, such a bipolar outcome is tempered if collaboration leads to a reduction of both expected unit cost and cost uncertainty. We investigate those cases next, starting with the special case we examined in Proposition 3, where the lower bound of the unit cost function is constant.

Proposition 6 *Suppose that Assumption 3 holds. Let $q^\dagger$ be the solution of $J(q) = 0$ in Proposition 3.*

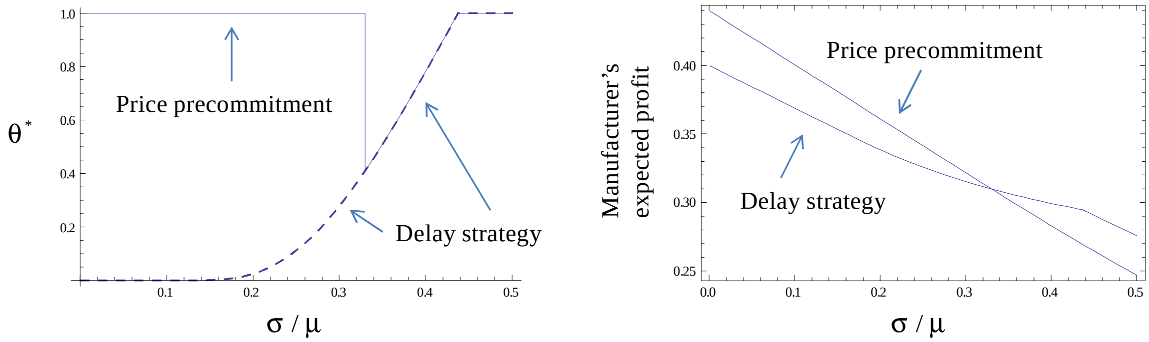
(i) If $q^\dagger \geq F^{-1}\left(1 - \frac{1}{r}(\underline{c} + 2\delta\Delta_0)\right)$, then $\pi_m^d > \pi_m^p$ and $\theta^d = 1$.

(ii) If $q^\dagger \leq F^{-1}\left(1 - \frac{1}{r}(\underline{c} + 2\Delta_0)\right)$, there is a unique $\widehat{\delta} \in (0,1)$ such that $\pi_m^d < \pi_m^p$ for $\delta < \widehat{\delta}$ and $\pi_m^d > \pi_m^p$ for $\delta > \widehat{\delta}$, while $\theta^d = 0$.

According to the proposition, the manufacturer prefers the delay strategy if the supplier chooses to fully collaborate under it. On the other hand, if the supplier refuses to collaborate under the same strategy, then price precommitment is better given that δ is sufficiently small. Combining this finding with the results in Proposition 4, we infer the following: the manufacturer prefers the delay strategy if cost uncertainty is not significantly reduced through collaboration and relatively large uncertainty in demand exists. Otherwise, she should opt for price precommitment. This insight is confirmed by numerical examples for cases with $0 < \theta^d < 1$. Figure 3 shows an example in which the delay strategy begins to dominate price precommitment as demand uncertainty exceeds a certain level.

This finding is in stark contrast to the conclusion from the benchmark case, in which the delay strategy was found to be optimal regardless of uncertainties of unit cost and demand. These environmental characteristics turn out to play important roles in influencing the supplier's incentive to collaborate when the supply chain is not integrated, which the manufacturer should take into account when she decides on a contracting strategy. In the end, the delay strategy may still be optimal but only in limited situations. Instead, price precommitment emerges as a superior alternative in many situations. An added benefit of the precommitment strategy is that contracts are easy to implement; there is no need to devise a sophisticated screening mechanism.

While capturing the essential insights, the results in Proposition 6 are somewhat incomplete since they rely on the specific assumption that the lower bound of the unit cost does not vary



(a) Optimal collaboration level at the equilibrium (b) The manufacturer's expected profit at the equilibrium

Figure 3: The optimal collaboration level and the manufacturer's expected profit at the equilibrium under the two strategies. In this example, $\delta = 0.9$ and normal demand distributrion are assumed.

with θ. Extensive numerical experiments show that our earlier conclusion continues to be true in more general cases as well. That is, (a) under the delay strategy, the optimal collaboration level increases in δ as well as in demand uncertainty, and (b) the manufacturer prefers the delay strategy if there is large demand uncertainty and collaboration does not lead to significant reduction of cost uncertainty.[7] See Figure 4 for a concise summary of our numerical results. In fact, observations from the numerical experiments allow us to make a more definitive statement: the delay strategy is more valued in those cases where the rate of reduction of the expected unit cost, $\gamma_0 - \gamma_1$, is much higher than the rate of cost uncertainty reduction, $(1-\delta)\Delta_0$. This is a direct consequence of the supplier's incentive to collaborate. Under such a condition, the supplier subject to the delay strategy can protect his private information effectively even after collaborating, since cost uncertainty remains significant. Thus, he is more willing to collaborate, and this voluntary action reduces the need for the manufacturer to take the costly risk of precommitting in order to induce the supplier to fully collaborate.

[7] Gilbert and Cvsa (2003) offer an insight similar in spirit to ours in one particular respect: they find that precommitment is preferred by the party who offers a wholesale contract when uncertainty of the demand curve is low. However, their result is neither driven by information asymmetry or cost uncertainty, which play paramount role in our analysis.

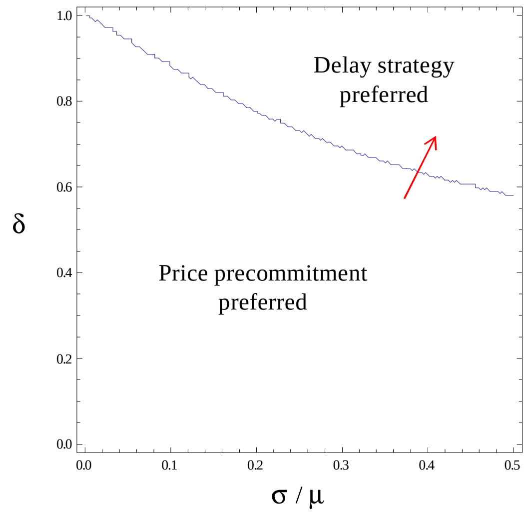


Figure 4: The regions in which one strategy dominates the other. The horizontal axis represents demand uncertainty and the vertical axis represents residual cost uncertainty. In this example, $r = 1$ and $\gamma_0 - \gamma_1 = 0.5$. The arrow indicates the direction of changes as $\gamma_0 - \gamma_1$ decreases.

6 Extension: Collaboration in the Make-to-Order Production Environment

So far, we have based our analysis on the assumption that production lead time is long, and therefore the manufacturer has to source the component well before demand is known. While this assumption is quite realistic for the complex products that we considered as a motivation, there may be situations where the lead time is relatively short. An obvious benefit of a short lead time is that demand-supply mismatch risk can be significantly reduced by quickly manufacturing the components and the end products after observing initial demands and updating the demand forecast (such as in the quick response strategy; Fisher and Raman 1996, Iyer and Bergen 1997). Naturally, lead time reduction is often cited as one of the main goals of supply chain management. Note that one way to achieve this goal is through supply chain collaboration during production (as opposed to collaboration during product design, which we have considered thus far). In this section, we investigate how lead time reduction in the production stage and the resulting improvement in matching supply with demand impact the collaboration decision in the product design stage. To illustrate our point, we make a simplifying assumption that the lead time is negligible so the supply chain turns into a make-to-order system. The following lemma shows what happens to the supplier's incentive to collaborate in such a circumstance.

Lemma 3 *Suppose that no demand-supply mismatch exists. In a decentralized supply chain:*

(i) Price precommitment strictly dominates price-quantity precommitment as well as quantity precommitment. The supplier chooses $\theta = 1$ under price precommitment.

(ii) Under the delay strategy, the supplier chooses $\theta = 0$.

Recall from Section 5.1 that price precommitment and price-quantity precommitment are equivalent when the quantity decision has to be made before demand uncertainty is resolved. Part (i) of the lemma shows that this symmetry breaks down in the absence of mismatch risk. This is because price precommitment is more flexible; the manufacturer can wait until demand is realized to determine the order quantity under price precommitment, whereas she cannot do so under price-quantity precommitment. We also find that, consistent with our earlier result, the supplier fully collaborates under price precommitment.

When the delay strategy is employed, on the other hand, the supplier does not collaborate which contrasts with what we found in Proposition 3, in which the optimal collaboration level could be anywhere between 0 and 1. The difference arises because perfect demand-supply matching under the delay strategy functions like quantity precommitment; the supplier expects that the order quantity will be equal to demand, whose mean is known. Therefore, the collaboration does not result in the volume increase benefit, and the supplier chooses not to collaborate in order to protect himself from disadvantageous pricing.

This observation suggests that a potential conflict may exist between collaboration during product design and collaboration during production. If the production lead time is reduced through production stage collaboration to the extent that the manufacturer can avoid demand-supply mismatch altogether (as suggested by the quick response system), the supplier is not incentivized to collaborate in the product design stage. That is, ex-post collaboration is at odds with ex-ante collaboration. One way to avoid such a conflict is to precommit to price. In fact, we find that the delay strategy should never be used in the current setting.

	Centralized supply chain	**Decentralized supply chain**
Make-to-stock	Delay strategy	Delay strategy or price/price-quantity precommitment
Make-to-order	Delay strategy	Price precommitment

Table 1: Optimal contracting strategies in different production environments.

Proposition 7 *Suppose that no demand-supply mismatch exists. In a centralized supply chain the integrated firm chooses the delay strategy, whereas in a decentralized supply chain, the manufacturer chooses the price precommitment strategy.*

In a decentralized make-to-order supply chain, price precommitment strictly dominates the delay strategy regardless of how unit cost function changes with collaboration or how uncertain the demand is. In addition, the contrast between the centralized and decentralized settings is at its extreme since the optimal strategies do not overlap. Therefore, we come to the following paradoxical conclusion: although short production lead time enables reactive capacity which is conducive to the delay strategy, in a decentralized supply chain, the precommitment strategy actually creates a higher value than does the delay strategy. Table 1 summarizes the optimal strategies in different production environments.

7 Conclusion

In this paper we study how information asymmetry and contracting strategies impact the effectiveness of supply chain collaboration. Despite a high level of interests among the practitioners of supplier management and strategic sourcing, the topic of supply chain collaboration has received relatively little attention in the OM literature. We aim to fill this gap by presenting a model that delves into one of the most important issues of collaboration, namely, firms' desire to balance the benefit of cooperation with the need to protect their proprietary information.

We begin with the premise that collaboration is most important during the early product design stage, in which there is a high level of uncertainty about the unit cost of production. Firms collaborate in order to learn each other's expectation and capability to achieve the mutual benefit of reduced expected cost and cost uncertainty. Building on the traditional frameworks of the newsvendor model and adverse selection, we develop a stylized two-period model in which the supplier's incentive to collaborate during the product design stage is determined by the presence of asymmetric information about the cost in the later production stage. We also investigate how environmental characteristics such as uncertainties in demand and cost influence this incentive. Considering various ways that unit cost distribution changes with the degree of collaboration, we obtain insights into which circumstances are conducive to the supplier engaging in collaboration and into when it is optimal for the manufacturer to offer a procurement contract. In particular, we compare precommitment strategies and the delay contracting strategy, and specify conditions under which the manufacturer prefers a particular strategy.

We find that, when the supply chain is integrated, a newsvendor firm always prefers to wait until cost uncertainty disappears before determining the optimal production quantity. In a decentralized supply chain, however, such a delay strategy creates an incentive issue. Although the manufacturer prefers the supplier to fully collaborate with her during the product design stage, the supplier may not agree to do so fearing that collaboration will reveal important information about the supplier's cost structure which the opportunistic manufacturer might take advantage of when offering the supplier a procurement contract. The optimal contract in this case has a screening form, which helps the manufacturer efficiently extract the supplier's surplus created by information asymmetry. If, on the other hand, the manufacturer commits to price before collaboration begins, the supplier will fully collaborate. However, the manufacturer's ability to extract surplus is quite limited in this case. Comparing these two contracting strategies, we find that the environment is suitable for the manufacturer to adopt the delay strategy when (a) collaboration brings a significant reduction of expected unit cost but not of cost uncertainty, and (b) large demand uncertainty is present. If these conditions are not satisfied, price precommitment is recommended. The value of precommitment is especially high when the firms can improve supply chain efficiency by reducing demand uncertainty and/or production lead time, since these actions, paradoxically, reduce the incentives to collaborate when the delay strategy is used. These results contrast with the well-established principle in the OM literature that, whenever possible, firms should postpone a decision until uncertainty diminishes. Our analysis shows that such a conclusion should be reevaluated when incentives are involved. In addition, our analysis provides evidence that in many practical situations a simple contracting approach such as price precommitment is more effective than a complex one such as a screening contract.

The insights generated from this model rest on a few specific assumptions which represent abstractions of reality. Hence, the implications of our model are to be understood in the context described in the paper. Relaxing some of these assumptions and including more real-world considerations into the model will bring additional analytical challenges but will undoubtedly enrich the managerial insights and present an opportunity to test the robustness of our findings. For example, we assumed that the only outcome of collaboration is changes in unit cost parameters, based on many industry reports that cost reduction is the most important reason firms enter into collaborative relationships. However, it is likely that other factors need to be considered simultaneously. Bhaskaran and Krishnan (2009) consider revenue sharing among collaboration partners and reduction of uncertainty regarding product development lead time. Wang et al. (2009) consider improvement of supplier reliability. Although the focus of this paper is specifically on the role of incentives in collaboration, a more complete theory of supply chain collaboration will emerge once these factors become integral parts of the model.

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Appendix

A Intuition Behind Proposition 4, part (ii)

Consider a supplier of a given type z, which is revealed to him at the end of the product design stage. Under Assumptions 2 and 3, the value of the unit cost $G^{-1}(z \mid \theta)$ and the spread around it are the largest for this supplier if he chooses $\theta = 0$. Suppose that he indeed chooses $\theta = 0$. At this level, the manufacturer has to charge a high price to ensure the supplier's participation since the unit cost is high. If this price is sufficiently high, overage risk is a bigger concern than underage risk to the manufacturer, who is a newsvendor. Hence, in this case, an increase in demand uncertainty prompts the manufacturer to reduce order quantity. Since this is true for all supplier types, the supplier who chooses $\theta = 0$ in the product design stage expects lower order quantity as demand uncertainty increases. At the same time, the manufacturer tries to compensate for the loss due to increased demand-supply mismatch by lowering the price to extract more from the supplier's profit margin, i.e., information rent per unit. Her ability to do so is most effective against the supplier who chooses $\theta = 0$, since the unit cost spread is widest at that level. Therefore, the net effect of increased demand uncertainty to the supplier who chooses $\theta = 0$ is reduction of expected profit, as both the order quantity and the profit margin are lowered.

By a similar argument, a supplier who chooses $\theta = 1$ expects to be offered a larger quantity in response to a demand uncertainty increase, since underage risk outweighs overage risk at this collaboration level where the unit cost is smallest for all z. However, the manufacturer's ability to extract more from this supplier's profit margin is limited since the unit cost spread is also small. Hence, when the changes in the quantity and the profit margin are combined, this supplier's expected profit may either increase or decrease slightly.

Comparing and generalizing what happen to these two suppliers, we can see that higher demand uncertainty shifts the supplier's profit function (which was shown in Proposition 3 to be quasiconcave in θ) in such a way that the profit function curve is significantly depressed in the low-θ region, while it is either raised or slightly lowered in the high-θ region (see Figure 5). As a result, the optimal θ that maximizes the supplier's profit moves from left to right, i.e., higher θ is chosen by the supplier.

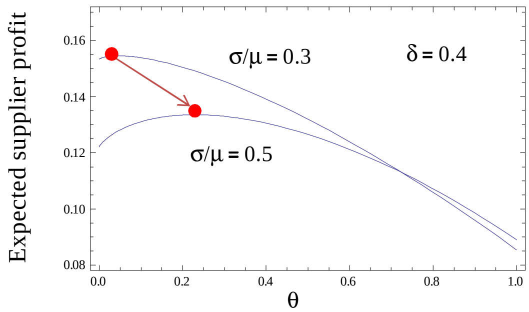

Figure 5: An illustration of how the expected supplier profit function changes as demand uncertainty increases, when a screening contract is offered in the production stage.

B Proofs

Proof of Proposition 1. First consider the precommitment strategy. It is clear from the profit expression in $(\mathcal{IP})$ that the integrated firm chooses $\theta = 1$, since Assumption 2 implies that $\frac{\partial}{\partial \theta} \int_0^1 G^{-1}(z \,|\, \theta) dz < 0$. Solving $(\mathcal{IP})$ for q with $\theta = 1$ yields the optimal solution $q_p^I = F^{-1}\left(1 - \frac{1}{r} \int_0^1 G^{-1}(z \,|\, 1) dz\right) = F^{-1}\left(1 - \frac{\gamma_1}{r}\right)$. Substituting this, the firm's expected profit at the optimum is $\Pi_p^I \equiv rE[\min\{D, q_p^I\}] - \left(\int_0^1 G^{-1}(z \,|\, 1)dz\right) q_p^I = r \int_0^{q_p^I} x f(x)dx + r q_p^I \overline{F}(q_p^I) - \gamma_1 q_p^I = r\mu(q_p^I)$.

Next, consider the delay strategy. From $(\mathcal{ID})$ we get $q(z \,|\, \theta) = F^{-1}\left(1 - \frac{1}{r} G^{-1}(z \,|\, \theta)\right)$. Substituting this, $rE[\min\{D, q(z \,|\, \theta)\}] - G^{-1}(z \,|\, \theta) q(z \,|\, \theta) = r\mu(q(z \,|\, \theta))$. Hence, the firm chooses θ that maximizes $\Pi_d(\theta) \equiv r \int_0^1 \mu(q(z \,|\, \theta))dz$. Consider the two cases in the proposition.

(i) The condition $\gamma_0 - \gamma_1 \geq (1 - \delta)\Delta_0/2$ implies $\frac{\partial}{\partial \theta} G^{-1}(z \,|\, \theta) \leq 0$ for all $z \in [0, 1]$, with the equality holding for $z = 0$ only (see Section 3.1). Hence, $\frac{\partial}{\partial \theta} q(z \,|\, \theta) \geq 0$ and $\Pi_d'(\theta) = \frac{\partial}{\partial \theta}\left(r \int_0^1 \mu(q(z \,|\, \theta))dz\right) = r \int_0^1 \frac{\partial}{\partial \theta} \mu(q(z \,|\, \theta))dz = r \int_0^1 q(z \,|\, \theta) f(q(z \,|\, \theta)) \frac{\partial}{\partial \theta} q(z \,|\, \theta)dz > 0$, where we used Leibniz's rule. Therefore, it is optimal for the firm to choose $\theta = 1$. Then the optimal quantity is $q_d^I(z) \equiv q(z \,|\, 1) = F^{-1}\left(1 - \frac{1}{r}\left[\gamma_1 + \delta\Delta_0\left(z - \frac{1}{2}\right)\right]\right)$ and the firms' expected profit at the optimum is $\Pi_d^I \equiv r \int_0^1 \mu(q_d^I(z))dz$. To find out which strategy leads to higher expected profit, examine the properties of $\chi(z) \equiv \mu(q_d^I(z)) - \mu(q_p^I)$. Since the last term is independent of z, $\chi'(z) = \frac{d}{dz}\mu(q_d^I(z)) = q_d^I(z) f(q_d^I(z)) \frac{d}{dz} q_d^I(z) = -\frac{\delta\Delta_0}{r} q_d^I(z) < 0$ and $\chi''(z) = -\frac{\delta\Delta_0}{r} \frac{d}{dz} q_d^I(z) > 0$. Therefore, $\chi(z)$ is a convex decreasing function. Notice also that $q_d^I(1/2) = q_p^I$, hence, $\chi(1/2) = 0$. Combining these two facts, we conclude that $\Pi_d^I - \Pi_p^I = r \int_0^1 \chi(z)dz > 0$, since convex decreasing $\chi(z)$ with $\chi(1/2) = 0$ implies

$\int_0^{1/2} \chi(z)dz > -\int_{1/2}^1 \chi(z)dz$. Therefore, $\Pi_d^I > \Pi_p^I$, i.e., it is optimal for the integrated firm to choose the delay strategy.

(ii) Let $\gamma_0 = \gamma_1$. Then $q(z\,|\,\theta) = F^{-1}\left(1 - \frac{1}{r}\left[\gamma_0 + \Delta_0\left(1-(1-\delta)\theta\right)\left(z-\frac{1}{2}\right)\right]\right)$ and $\Pi_d'(\theta) = r\int_0^1 \frac{\partial}{\partial\theta}\mu(q(z\,|\,\theta))dz = r\int_0^1 q(z\,|\,\theta)f(q(z\,|\,\theta))\frac{\partial}{\partial\theta}q(z\,|\,\theta)dz = (1-\delta)\Delta_0\int_0^1\left(z-\frac{1}{2}\right)q(z\,|\,\theta)dz$. Integrating by parts,

$$\begin{aligned}\int_0^1\left(z-\frac{1}{2}\right)q(z\,|\,\theta)dz &= \frac{1}{2}(z^2-z)q(z\,|\,\theta)\Big|_{z=0}^{z=1} - \frac{1}{2}\int_0^1 (z^2-z)\frac{\partial q(z\,|\,\theta)}{\partial z}dz \\ &= \frac{\Delta_0\left(1-(1-\delta)\theta\right)}{2r}\int_0^1 \frac{z^2-z}{f(q(z\,|\,\theta))}dz < 0,\end{aligned}$$

since $z^2 - z < 0$ for all $z \in (0,1)$. Therefore, $\Pi_d'(\theta) < 0$ and it is optimal for the firm to choose $\theta = 0$. Then the optimal quantity is $q_d^I(z) \equiv q(z\,|\,0) = F^{-1}\left(1-\frac{1}{r}\left[\gamma_0 + \Delta_0\left(z-\frac{1}{2}\right)\right]\right)$. The rest of the analysis, i.e., a comparison between the two strategies, is similar to that of (i) and leads to the same conclusion: the delay strategy is optimal.

■

Proof of Lemma 1. Under all strategies, the optimal contract consists of a single price w and a single quantity q since the screening mechanism does not apply when (a) only one of the two contract parameters is offered at a time or (b) no information asymmetry exists. It is straightforward to prove that the manufacturer's expected profit is maximized at $\theta = 1$ in all three cases.

Suppose that the manufacturer precommits q and offers w at the beginning of the production stage. At the latter point in time, the supplier has chosen θ. The manufacturer faces the problem

$$\max_w \; rE[\min\{D,q\}] - wq \;\; \text{subject to } (w - G^{-1}(z\,|\,\theta))q \geq 0,\, \forall z \in [0,1].$$

The solution is $w = G^{-1}(1\,|\,\theta)$, i.e., the manufacturer chooses a price that leaves zero profit to the supplier with the highest cost. This ensures the participation constraint. Anticipating this pricing, the supplier chooses the optimal collaboration level to maximize his expected profit $\int_0^1 (w - G^{-1}(z\,|\,\theta))qdz = \int_0^1 [G^{-1}(1\,|\,\theta) - G^{-1}(z\,|\,\theta)]qdz$. Since $G^{-1}(1\,|\,\theta) - G^{-1}(z\,|\,\theta) = \Delta_0\left(1-(1-\delta)\theta\right)(1-z)$ decreases in θ, the expected profit is maximized at $\theta = 0$.

Suppose that the manufacturer precommits w and offers q at the beginning of the production stage. At the latter point in time, the manufacturer chooses $q = F^{-1}\left(1 - \frac{w}{r}\right)$ to maximize her profit $rE[\min\{D, q\}] - wq$. Anticipating this, the supplier chooses the optimal collaboration level to maximize his expected profit $\int_0^1 (w - G^{-1}(z \mid \theta))qdz = \left(w - \int_0^1 G^{-1}(z \mid \theta)dz\right) F^{-1}\left(1 - \frac{w}{r}\right)$. Since $\int_0^1 G^{-1}(z \mid \theta)dz$ decreases in θ by Assumption 2, it is optimal for the supplier to choose $\theta = 1$. Now suppose that the manufacturer precommits both w and q. The supplier's expected profit in (IC-P) of the problem $(\mathcal{P})$ is $\int_0^1 (w - G^{-1}(z \mid \theta))qdz = \left(w - \int_0^1 G^{-1}(z \mid \theta)dz\right) q$. Again, the supplier maximizes his expected profit by choosing $\theta = 1$. ∎

Proof of Proposition 2.

Under quantity precommitment, the optimal price is $w_1 = G^{-1}(1 \mid 0)$ (see the proof of Lemma 1). The manufacturer then chooses q that maximizes her expected profit $rE[\min\{D, q\}] - w_1 q$, i.e., $q_1 = F^{-1}\left(1 - \frac{1}{r}\left(\gamma_0 + \frac{\Delta_0}{2}\right)\right)$. The manufacturer's expected profit at the equilibrium is then $r\mu(q_1)$.

Under price commitment, the optimal quantity is $q_2 = F^{-1}\left(1 - \frac{w}{r}\right)$ and the supplier chooses $\theta = 1$. The manufacturer then chooses w that maximizes her expected profit $rE[\min\{D, q_2\}] - wq_2 = r\mu(q_2)$ subject to the participation constraint $(w - G^{-1}(z \mid 1))q_2 \geq 0$, $\forall z \in [0, 1]$, which is equivalent to $(w - G^{-1}(1 \mid 1))q_2 = \left(w - \gamma_1 - \frac{\delta\Delta_0}{2}\right) q_2 \geq 0$. It can be shown that the left-hand side of this constraint is nonnegative and quasiconcave for $w \in [\gamma_1 + \delta\Delta_0/2, r)$. Since q_2 decreases in w and hence the manufacturer's profit decreases in w, the manufacturer maximizes her profit by reducing w until the supplier's profit hits zero at $w_2 = G^{-1}(1 \mid 1) = \gamma_1 + \delta\Delta_0/2$. Hence, $q_2 = F^{-1}\left(1 - \frac{1}{r}\left(\gamma_1 + \frac{\delta\Delta_0}{2}\right)\right)$ and the manufacturer's expected profit at the equilibrium is then $r\mu(q_2)$. It can be easily verified that the same solution is obtained under price-quantity commitment.

Since $\left(\gamma_0 + \frac{\Delta_0}{2}\right) - \left(\gamma_1 + \frac{\delta\Delta_0}{2}\right) = (\gamma_0 - \gamma_1) + \frac{(1-\delta)\Delta_0}{2} > 0$, $q_1 < q_2$. Therefore, $r\mu(q_1) < r\mu(q_2)$, i.e., the manufacturer strictly prefers price commitment and price-quantity commitment to quantity commitment. ∎

Proof of Lemma 2. The proof closely follows the approaches found in Ha (2001) and Bolton and Dewatripont (2005). Let $\nu(z \mid \theta) \equiv z/g(G^{-1}(z \mid \theta) \mid \theta)$, which is equal to $\Delta_0 (1 - (1 - \delta)\theta) z$ under (1). Since θ is fixed at the time the manufacturer decides the contract terms, we drop it in the subsequent expressions for notational convenience. Let $t(z) \equiv w(z)q(z)$ be the total payment to the z-type supplier. Once the optimal $t^*(z)$ and $q^*(z)$ are found, $w^*(z)$ is computed as $w^*(z) = t^*(z)/q^*(z)$. The expected profits are rewritten as $\pi_s(z, \widehat{z}) = t(\widehat{z}) - G^{-1}(z)q(\widehat{z})$ and $\pi_m = \int_0^1 (rE[\min\{D, q(z)\}] - t(z))\, dz$. For brevity, let $\pi_s(z) \equiv \pi_s(z, z)$.

First, we establish that (IR-D) can be rewritten as $\pi_s(1) = 0$. To see this, observe that (IC-D) and $G^{-1}(z) \leq G^{-1}(1)$ together imply that $\pi_s(z) = t(z) - G^{-1}(z)q(z) \geq t(1) - G^{-1}(z)q(1) \geq t(1) - G^{-1}(1)q(1) = \pi_s(1)$. Therefore, $\pi_s(1) \geq 0$ guarantees (IR-D). Suppose $\pi_s(1) > 0$ at the optimum. By reducing $t(z)$ by the same infinitesimal amount for all z, neither (IR-D) or (IC-D) is violated while π_m is increased. Therefore, $\pi_s(1) = 0$ at the optimum.

Next, we show that (IC-D) is equivalent to the two conditions

$$t'(z) = G^{-1}(z)q'(z), \tag{4}$$

$$q'(z) \leq 0. \tag{5}$$

Observe that (IC-D) implies the first- and second-order conditions $\frac{\partial}{\partial \widehat{z}}\pi_s(z, \widehat{z})\big|_{\widehat{z}=z} = 0$ and $\frac{\partial^2}{\partial \widehat{z}^2}\pi_s(z, \widehat{z})\Big|_{\widehat{z}=z} \leq 0$. (4) follows from the first-order condition. The second-order condition is written as $t''(z) - G^{-1}(z)q''(z) \leq 0$. On the other hand, differentiating (4) yields $t''(z) - \frac{q'(z)}{g(G^{-1}(z))} - G^{-1}(z)q''(z) = 0$. (5) follows from combining these two results. Therefore, (IC-D) implies (4) and (5). Conversely, suppose that (4) and (5) are true. Assume that there exists z that violates (IC-D), i.e., $\pi_s(z) = t(z) - G^{-1}(z)q(z) < t(\widehat{z}) - G^{-1}(z)q(\widehat{z}) = \pi_s(z, \widehat{z})$, or equivalently, $\int_z^{\widehat{z}} \left[t'(x) - G^{-1}(z)q'(x)\right] dx > 0$. Consider $z < \widehat{z}$. From (5), we have $G^{-1}(z)q'(x) \geq G^{-1}(x)q'(x)$ for $x \in [z, \widehat{z}]$. Then $\int_z^{\widehat{z}} \left[t'(x) - G^{-1}(z)q'(x)\right] dx \leq \int_z^{\widehat{z}} \left[t'(x) - G^{-1}(x)q'(x)\right] dx = 0$ by (4). But this contradicts the earlier assertion $\int_z^{\widehat{z}} \left[t'(x) - G^{-1}(z)q'(x)\right] dx > 0$. A similar argument can be made for $z \geq \widehat{z}$.

Summarizing, we have replaced (IR-D) with $\pi_s(1) = 0$ and (IC-D) with $t'(z) = G^{-1}(z)q'(z)$ and $q'(z) \leq 0$. To solve this modified optimization problem, we ignore the last constraint $q'(z) \leq 0$ and solve the relaxed problem instead, and then verify that the omitted constraint is indeed satisfied. By (4), we have $\pi_s'(z) = -\frac{q(z)}{g(G^{-1}(z))}$. Integrating both sides of this equation and using $\pi_s(1) = 0$, we obtain $\pi_s(z) = \int_z^1 \frac{q(x)}{g(G^{-1}(x))} dx$. Therefore, $t(z) = G^{-1}(z)q(z) + \pi_s(z) = G^{-1}(z)q(z) + \int_z^1 \frac{q(x)}{g(G^{-1}(x))} dx$, or equivalently, $w(z) = \frac{t(z)}{q(z)} = G^{-1}(z) + \frac{1}{q(z)} \int_z^1 \frac{q(x)}{g(G^{-1}(x))} dx$. As a result, the manufacturer's expected profit becomes

$$\pi_m = \int_0^1 \left(rE[\min\{D, q(z)\}] - G^{-1}(z)q(z) - \int_z^1 \frac{q(x)}{g(G^{-1}(x))} dx \right) dz.$$

Observe that, by integration by parts,

$$\int_0^1 \left(\int_z^1 \frac{q(x)}{g(G^{-1}(x))}dx\right) dz = \left[\left(\int_z^1 \frac{q(x)}{g(G^{-1}(x))}dx\right) z\right]_0^1 + \int_0^1 \frac{zq(z)}{g(G^{-1}(z))}dz = \int_0^1 \nu(z)q(z)dz. \quad (6)$$

Hence, $\pi_m = \int_0^1 \left(rE[\min\{D, q(z)\}] - \left(G^{-1}(z) + \nu(z)\right) q(z)\right) dz$. Differentiating this with respect to $q(z)$, $\partial\pi_m/\partial q(z) = r\overline{F}(q(z)) - \left(G^{-1}(z) + \nu(z)\right)$ and $\partial^2\pi_m/\partial q(z)^2 = -rf(q(z)) < 0$. Hence, the manufacturer's problem is concave. Suppose that $q(z) = 0$ at the optimum. Then $\partial\pi_m/\partial q(z) \le 0$ at $q(z) = 0$, i.e., $G^{-1}(z) + \nu(z) \ge r$. However, this contradicts the earlier assumption $G^{-1}(z \,|\, \theta) + \nu(z \,|\, \theta) < r$. Therefore, $q(z) > 0$ at the optimum. Noting that $\lim_{q(z)\to\infty} \partial\pi_m/\partial q(z) = -\left(G^{-1}(z) + \nu(z)\right) < 0$, we conclude that the optimal $q(z)$ is found from the first-order condition $\partial\pi_m/\partial q(z) = 0$, i.e., $q^*(z) = F^{-1}\left(1 - \frac{1}{r}\left[G^{-1}(z) + \nu(z)\right]\right)$. Finally, $\partial q^*(z)/\partial z < 0$ since $G^{-1}(z) + \nu(z)$ is increasing, confirming the condition (5) that was left out earlier.

The supplier's expected profit is $\pi_s^* = \int_0^1 \pi_s^*(z)dz = \int_0^1 \left(\int_z^1 \frac{q^*(x)}{g(G^{-1}(x))}dx\right) dz = \int_0^1 \nu(z)q^*(z)dz$, where we used (6). The manufacturer's expected profit, after substituting $q^*(z)$ in her objective function, is $\pi_m^* = r\int_0^1 \mu(q^*(z))dz$. ■

Proof of Proposition 3. It is straightforward to prove that the manufacturer's expected profit $\pi_m^*(\theta)$ is an increasing function. Differentiating $\pi_s^*(\theta)$ found in Lemma 2,

$$\frac{d\pi_s^*(\theta)}{d\theta} = -\Delta_0(1-\delta)\int_0^1 zq^*(z \,|\, \theta)dz + \Delta_0\left(1-(1-\delta)\theta\right)\int_0^1 z\frac{\partial q^*(z \,|\, \theta)}{\partial\theta}dz.$$

It is straightforward to verify

$$\frac{\partial q^*(z \,|\, \theta)}{\partial\theta} = -\frac{1-\delta}{1-(1-\delta)\theta} z\frac{\partial q^*(z \,|\, \theta)}{\partial z}. \quad (7)$$

Observe that, using (7) and integration by parts, the second term becomes

$$\begin{aligned}
&\Delta_0\left(1-(1-\delta)\theta\right)\int_0^1 z\frac{\partial q^*(z \,|\, \theta)}{\partial\theta}dz \\
=\;& -\Delta_0(1-\delta)\int_0^1 z^2\frac{\partial q^*(z \,|\, \theta)}{\partial z}dz = -\Delta_0(1-\delta)\left(z^2q^*(z \,|\, \theta)\big|_{z=0}^{z=1} - 2\int_0^1 zq^*(z \,|\, \theta)dz\right) \\
=\;& -\Delta_0(1-\delta)q^*(1 \,|\, \theta) + 2\Delta_0(1-\delta)\int_0^1 zq^*(z \,|\, \theta)dz.
\end{aligned}$$

Hence,

$$\frac{d\pi_s^*(\theta)}{d\theta} = \Delta_0(1-\delta)\left(-q^*(1\,|\,\theta) + \int_0^1 zq^*(z\,|\,\theta)dz\right). \tag{8}$$

Inverting $q^*(z\,|\,\theta) = F^{-1}\left(1 - \frac{1}{r}\left[\underline{c} + 2\Delta_0\left(1-(1-\delta)\theta\right)z\right]\right)$ found in Lemma 2,

$$z = \frac{r\overline{F}(q) - \underline{c}}{2\Delta_0\left(1-(1-\delta)\theta\right)} \text{ and } dz = \frac{-rf(q)}{2\Delta_0\left(1-(1-\delta)\theta\right)}dq.$$

Therefore,

$$\int_0^1 zq^*(z\,|\,\theta)dz = \frac{r^2}{4\Delta_0^2\left(1-(1-\delta)\theta\right)^2}\int_{q^*(1\,|\,\theta)}^{q^*(0\,|\,\theta)} (\overline{F}(q) - \underline{c}/r)qf(q)dq.$$

Hence, the first-order condition $d\pi_s^*(\theta)/d\theta = 0$, or, $q^*(1\,|\,\theta) = \int_0^1 zq^*(z\,|\,\theta)dz$ becomes

$$q^*(1\,|\,\theta) = \frac{r^2}{4\Delta_0^2\left(1-(1-\delta)\theta\right)^2}\int_{q^*(1\,|\,\theta)}^{q^*(0\,|\,\theta)} (\overline{F}(q) - \underline{c}/r)qf(q)dq.$$

Since $q^*(1\,|\,\theta)$ satisfies $r\overline{F}(q^*(1\,|\,\theta)) = \underline{c} + 2\Delta_0\left(1-(1-\delta)\theta\right)$, this is equal to the following relation for $q = q^*(1\,|\,\theta)$:

$$q = \frac{1}{\left(\overline{F}(q) - \underline{c}/r\right)^2}\int_q^{F^{-1}(1-\underline{c}/r)} (\overline{F}(x) - \underline{c}/r)xf(x)dx.$$

Notice that we have transformed the equation in θ into the equation in q. This relation is equivalent to $J(q) = 0$ for $q < F^{-1}\left(1 - \underline{c}/r\right)$. Once $q^\dagger$ that satisfies this equation is computed, then the optimal θ is found from $r\overline{F}(q^\dagger) = \underline{c} + 2\Delta_0\left(1-(1-\delta)\theta\right)$, or (3).

Next, we prove that there is a unique $q^\dagger < F^{-1}\left(1-\underline{c}/r\right)$ that satisfies $J(q) = 0$. Taking a derivative, $dJ(q)/dq = \xi_1(q)\xi_2(q)$, where $\xi_1(q) \equiv \overline{F}(q) - qf(q) - \underline{c}/r$ and $\xi_2(q) \equiv \overline{F}(q) - \underline{c}/r$. Let q_1 and q_2 be the roots of $\xi_1(q)$ and $\xi_2(q)$, respectively. Recall that $q_2 = F^{-1}\left(1-\underline{c}/r\right)$ is the upper bound on q. Since F has the IGFR property, $\overline{F}(q) - qf(q) < \overline{F}(q)$ in $\xi_1(q)$ is decreasing and, as a result, $q_1 < q_2$, if q_1 exists. The existence is confirmed from the continuity of $\xi_1(q)$ along with $\xi_1(0) = 1 - \underline{c}/r > 0$ and $\lim_{q\to q_2}\xi_1(q) = -q_2 f(q_2) < 0$. Moreover, there is at most one q_1 satisfying $\xi_1(q_1) = 0$, as proved in Theorem 1 of Lariviere and Porteus (2001). We therefore conclude that there is a unique $q_1 < q_2$ such that $dJ(q)/dq|_{q=q_1} = 0$. Now observe that: (i) $J(0) = -\int_0^{q_2}(\overline{F}(x) - \underline{c}/r)xf(x)dx < 0$, (ii) $\lim_{q\to q_2} J(q) = 0$, (iii) $dJ(q)/dq|_{q=0} = (1-\underline{c}/r)^2 > 0$, and (iv) $\lim_{q\to q_2} dJ(q)/dq = 0$. In other words, $J(q)$ initially starts from a negative value with a positive slope, then converges to 0 with zero slope toward the domain upper bound q_2. Given our earlier result that there is a unique $q_1 < q_2$ such that $dJ(q)/dq|_{q=q_1} = 0$, the only possibility is that $J(q)$, starting from a negative value, increases until q reaches q_1 and then decreases to zero, implying that $J(q)$ crosses zero exactly once at $q^\dagger < q_1$.

Recall from (8) that $q^\dagger$ satisfies $q^\dagger = \int_0^1 zq^*(z \,|\, \theta^d)dz = \int_0^1 zF^{-1}\left(1 - \frac{1}{r}\left[\underline{c} + 2\Delta_0\left(1 - (1-\delta)\theta^d\right)z\right]\right)dz$. Suppose that $q^\dagger \le F^{-1}\left(1 - \frac{1}{r}(\underline{c} + 2\Delta_0)\right)$. If the supplier does not choose $\theta^d = 0$, i.e., if his expected profit is maximized at $\theta^d > 0$, $q^\dagger = \int_0^1 zF^{-1}\left(1 - \frac{1}{r}\left[\underline{c} + 2\Delta_0\left(1 - (1-\delta)\theta^d\right)z\right]\right)dz > \int_0^1 zF^{-1}\left(1 - \frac{1}{r}(\underline{c} + 2\Delta_0 z)\right)dz$. Then from (8),

$$\begin{aligned} \frac{1}{\Delta_0(1-\delta)} \frac{d\pi_s^*(\theta)}{d\theta}\bigg|_{\theta=0} &= -F^{-1}\left(1 - \frac{\underline{c} + 2\Delta_0}{r}\right) + \int_0^1 zF^{-1}\left(1 - \frac{\underline{c} + 2\Delta_0 z}{r}\right)dz \\ &\le -q^\dagger + \int_0^1 zF^{-1}\left(1 - \frac{\underline{c} + 2\Delta_0 z}{r}\right)dz < 0, \end{aligned}$$

implying that there is a local minimum of $\pi_s^*(\theta)$ between $\theta = 0$ and $\theta = \theta^d$. But this contradicts our earlier conclusion that there is a unique solution to the first-order condition $J(q) = 0$, or equivalently, $d\pi_s^*(\theta)/d\theta = 0$. Hence, $\theta^d = 0$ if $q^\dagger \le F^{-1}\left(1 - \frac{1}{r}(\underline{c} + 2\Delta_0)\right)$. Similar arguments prove that $\theta^d = 1$ if $q^\dagger \ge F^{-1}\left(1 - \frac{1}{r}(\underline{c} + 2\delta\Delta_0)\right)$ and $0 < \theta^d < 1$ if $F^{-1}\left(1 - \frac{1}{r}(\underline{c} + 2\Delta_0)\right) < q^\dagger < F^{-1}\left(1 - \frac{1}{r}(\underline{c} + 2\delta\Delta_0)\right)$. ■

Proof of Proposition 4. Part (i) follows directly from the expression (3). To prove part (ii), let Φ and ϕ be the cdf and the pdf of the standard normal distribution. (2) and (3) become

$$J(q^\dagger) = q^\dagger\left(\overline{\Phi}\left(\frac{q^\dagger - \mu}{\sigma}\right) - \frac{\underline{c}}{r}\right)^2 - \int_{\frac{q^\dagger - \mu}{\sigma}}^{\Phi^{-1}(1-\underline{c}/r)} \left(\overline{\Phi}(y) - \frac{\underline{c}}{r}\right)(\mu + \sigma y)\phi(y)\,dy, \tag{9}$$

$$\theta^d = \frac{\underline{c} + 2\Delta_0 - r\overline{\Phi}\left(\frac{q^\dagger - \mu}{\sigma}\right)}{2\Delta_0(1-\delta)}, \tag{10}$$

where we have let $y = (x - \mu)/\sigma$ inside the integral in (9). Implicit differentiation of the first-order condition $J(q^\dagger) = 0$ with respect to σ yields

$$\begin{aligned} 0 &= \left(\frac{\partial q^\dagger}{\partial \sigma}\right)\left(\overline{\Phi}\left(\frac{q^\dagger-\mu}{\sigma}\right) - \frac{\underline{c}}{r}\right)^2 - q^\dagger\left(\frac{\partial q^\dagger}{\partial \sigma} - \frac{q^\dagger-\mu}{\sigma}\right)\frac{1}{\sigma}\phi\left(\frac{q^\dagger-\mu}{\sigma}\right)\left(\overline{\Phi}\left(\frac{q^\dagger-\mu}{\sigma}\right) - \frac{\underline{c}}{r}\right) \\ &\quad - \int_{\frac{q^\dagger-\mu}{\sigma}}^{\Phi^{-1}(1-\underline{c}/r)} \left(\overline{\Phi}(y) - \frac{\underline{c}}{r}\right) y\phi(y)\,dy \\ &= \left(\frac{\partial q^\dagger}{\partial \sigma} - \frac{q^\dagger-\mu}{\sigma}\right)\left(\overline{\Phi}\left(\frac{q^\dagger-\mu}{\sigma}\right) - \frac{\underline{c}}{r}\right)\left(\overline{\Phi}\left(\frac{q^\dagger-\mu}{\sigma}\right) - \frac{\underline{c}}{r} - \frac{q^\dagger}{\sigma}\phi\left(\frac{q^\dagger-\mu}{\sigma}\right)\right) \\ &\quad + \frac{q^\dagger-\mu}{\sigma}\left(\overline{\Phi}\left(\frac{q^\dagger-\mu}{\sigma}\right) - \frac{\underline{c}}{r}\right)^2 - \int_{\frac{q^\dagger-\mu}{\sigma}}^{\Phi^{-1}(1-\underline{c}/r)} \left(\overline{\Phi}(y) - \frac{\underline{c}}{r}\right) y\phi(y)\,dy. \end{aligned}$$

Rewriting the first-order condition $J(q^\dagger) = 0$, the last integral is expressed as

$$\int_{\frac{q^\dagger-\mu}{\sigma}}^{\Phi^{-1}(1-\underline{c}/r)} \left(\overline{\Phi}(y) - \frac{\underline{c}}{r}\right) y\phi(y)\,dy = \frac{q^\dagger}{\sigma}\left(\overline{\Phi}\left(\frac{q^\dagger-\mu}{\sigma}\right) - \frac{\underline{c}}{r}\right)^2 - \frac{\mu}{\sigma}\int_{\frac{q^\dagger-\mu}{\sigma}}^{\Phi^{-1}(1-\underline{c}/r)} \left(\overline{\Phi}(y) - \frac{\underline{c}}{r}\right) \phi(y)\,dy.$$

Then

$$\begin{aligned} 0 &= \left(\frac{\partial q^\dagger}{\partial \sigma} - \frac{q^\dagger-\mu}{\sigma}\right)\left(\overline{\Phi}\left(\frac{q^\dagger-\mu}{\sigma}\right) - \frac{\underline{c}}{r}\right)\left(\overline{\Phi}\left(\frac{q^\dagger-\mu}{\sigma}\right) - \frac{\underline{c}}{r} - \frac{q^\dagger}{\sigma}\phi\left(\frac{q^\dagger-\mu}{\sigma}\right)\right) \\ &\quad - \frac{\mu}{\sigma}\left[\left(\overline{\Phi}\left(\frac{q^\dagger-\mu}{\sigma}\right) - \frac{\underline{c}}{r}\right)^2 - \int_{\frac{q^\dagger-\mu}{\sigma}}^{\Phi^{-1}(1-\underline{c}/r)} \left(\overline{\Phi}(y) - \frac{\underline{c}}{r}\right) \phi(y)\,dy\right]. \end{aligned}$$

The following can be shown by integration by parts:

$$\int_a^b \phi(y)\,\Phi(y)\,dy = \frac{1}{2}\left(\Phi(b)^2 - \Phi(a)^2\right).$$

Using this relation and after a few steps of algebra,

$$\int_{\frac{q^\dagger-\mu}{\sigma}}^{\Phi^{-1}(1-\underline{c}/r)} \left(\overline{\Phi}(y) - \frac{\underline{c}}{r}\right) \phi(y)\,dy = \frac{1}{2}\left(\overline{\Phi}\left(\frac{q^\dagger-\mu}{\sigma}\right) - \frac{\underline{c}}{r}\right)^2.$$

Substituting this back into the first-order equation, we obtain

$$\frac{\partial q^\dagger}{\partial \sigma} - \frac{q^\dagger-\mu}{\sigma} = \frac{\mu}{2\sigma}\frac{\xi_2(q^\dagger)}{\xi_1(q^\dagger)}, \tag{11}$$

where $\xi_1(q) \equiv \overline{F}(q) - qf(q) - \underline{c}/r = \overline{\Phi}\left(\frac{q-\mu}{\sigma}\right) - \frac{q}{\sigma}\phi\left(\frac{q-\mu}{\sigma}\right) - \frac{\underline{c}}{r}$ and $\xi_2(q) \equiv \overline{F}(q) - \underline{c}/r = \overline{\Phi}\left(\frac{q-\mu}{\sigma}\right) - \frac{\underline{c}}{r}$. Recall from the proof of Proposition 3 that $q^\dagger < q_1$, where q_1 satisfies $\xi_1(q) = 0$. Because of the IGFR property of F, it was shown that q_1 is a unique solution of $\xi_1(q) = 0$. Note also that $\xi_1(0) = 1 - \underline{c}/r > 0$, implying that $\xi_1(q) > 0$ for $q < q_1$. Hence, $\xi_1(q^\dagger) > 0$. In addition, it was also shown in the same proof that $q^\dagger < q_2$ where q_2 satisfies $\xi_2(q) = 0$. Since $\xi_2(q)$ is a decreasing function, $\xi_2(q^\dagger) > 0$. In sum, we have $\xi_1(q^\dagger) > 0$ and $\xi_2(q^\dagger) > 0$. Finally, differentiating (10) yields

$$\frac{\partial \theta^d}{\partial \sigma} = \frac{r}{2\Delta_0(1-\delta)} \left(\frac{\partial q^\dagger}{\partial \sigma} - \frac{q^\dagger - \mu}{\sigma} \right) \frac{1}{\sigma} \phi \left(\frac{q^\dagger - \mu}{\sigma} \right) = \frac{r}{2\Delta_0(1-\delta)} \frac{\mu}{2\sigma^2} \frac{\xi_2(q^\dagger)}{\xi_1(q^\dagger)} \phi \left(\frac{q^\dagger - \mu}{\sigma} \right) > 0,$$

where we used (11). ■

Proof of Proposition 5. It follows easily from the proof of Proposition 2 that the supplier chooses $\theta = 1$ and $q^p = F^{-1}\left(1 - \frac{1}{r}\left(\gamma_1 + \frac{\Delta_0}{2}\right)\right)$ in the limit $\delta \to 1$ under the precommitment strategy. The corresponding expected profits of the manufacturer and the supplier are $\pi_m^p = r\mu(q^p)$ and $\pi_s^p = \frac{1}{2}\delta\Delta_0 q^p$. Under the delay strategy, the optimal order quantity with the unit cost function (1) can be shown to be $q^*(z \,|\, \theta) = F^{-1}\left(1 - \frac{1}{r}\left[\gamma_0 - (\gamma_0 - \gamma_1)\theta + \Delta_0\left(1 - (1-\delta)\theta\right)\left(2z - \frac{1}{2}\right)\right]\right)$. The corresponding expected profits are $\pi_m^*(\theta) = r\int_0^1 \mu(q^*(z \,|\, \theta))dz$ and $\pi_s^*(\theta) = \Delta_0\left(1 - (1-\delta)\theta\right)\int_0^1 zq^*(z \,|\, \theta)dz$. Observe that $\lim_{\delta\to 1} q^*(z \,|\, \theta) = F^{-1}\left(1 - \frac{1}{r}\left[\gamma_0 - (\gamma_0 - \gamma_1)\theta + \Delta_0\left(2z - \frac{1}{2}\right)\right]\right)$ increases in θ. As a result, $\lim_{\delta\to 1} \pi_s^*(\theta) = \Delta_0 \int_0^1 zq^*(z \,|\, \theta)dz$ also increases in θ. Therefore, it is optimal for the supplier to choose $\theta = 1$. The comparison between the manufacturer's expected profits under the two cases proceeds similarly to that of Proposition 6. ■

Proof of Proposition 6. From Lemma 2 and Propositions 3 and 2, $\pi_m^d = r\int_0^1 \mu\left(q^*(z \,|\, \theta^d)\right)dz$ and $\pi_m^p = r\mu(q^p)$, where $q^*(z \,|\, \theta) = F^{-1}\left(1 - \frac{1}{r}\left[\underline{c} + 2\Delta_0\left(1 - (1-\delta)\theta\right)z\right]\right)$ and $q^p = F^{-1}\left(1 - \frac{1}{r}(\underline{c} + \delta\Delta_0)\right)$. Define $\chi_\theta(z) \equiv \mu\left(q^*(z \,|\, \theta)\right) - \mu(q^p)$. Observe that

$$\begin{aligned} \chi_\theta'(z) &= \frac{\partial}{\partial z}\left(\int_0^{q^*(z \,|\, \theta)} xf(x)dx - \int_0^{q^p} xf(x)dx\right) = q^*(z \,|\, \theta)f(q^*(z \,|\, \theta))\frac{\partial q^*(z \,|\, \theta)}{\partial z} \\ &= q^*(z \,|\, \theta)f(q^*(z \,|\, \theta))\left(-\frac{2\Delta_0\left(1 - (1-\delta)\theta\right)}{rf(q^*(z \,|\, \theta))}\right) = -\frac{2\Delta_0\left(1 - (1-\delta)\theta\right)}{r}q^*(z \,|\, \theta) < 0, \end{aligned}$$

and $\chi_\theta''(z) > 0$, which follows from the fact that $q^*(z \,|\, \theta)$ decreases in z. Therefore, $\chi_\theta(z)$ is a convex decreasing function.

(i) According to Proposition 3, $\theta^d = 1$ if $q^\dagger \geq F^{-1}\left(1 - \frac{1}{r}(\underline{c} + 2\delta\Delta_0)\right)$. Hence, $q^*\left(z \,|\, \theta^d = 1\right) = F^{-1}\left(1 - \frac{1}{r}(\underline{c} + 2\delta\Delta_0 z)\right)$. As shown above, $\chi_1(z)$ is convex decreasing. Notice also that

$\chi_1(1/2) = 0$. Combining these two facts, we conclude that $\pi_m^d - \pi_m^p = r\int_0^1 \chi_1(z)dz > 0$, since convexity of $\chi_1(z)$ implies $\int_0^{1/2} \chi_1(z)dz > -\int_{1/2}^1 \chi_1(z)dz$.

(ii) According to Proposition 3, $\theta^d = 0$ if $q^\dagger \leq F^{-1}\left(1 - \frac{1}{r}(\underline{c} + 2\Delta_0)\right)$. Hence, $q^*\left(z \,|\, \theta^d = 0\right) = F^{-1}\left(1 - \frac{1}{r}(\underline{c} + 2\Delta_0 z)\right)$, which is independent of δ. As before, $\chi_0(z)$ is convex decreasing. Since $\chi_0(\delta/2) = 0$, $\chi_0(z) > 0$ for $z < \delta/2$ and $\chi_0(z) < 0$ for $z > \delta/2$. Therefore, $\pi_m^d - \pi_m^p = r\int_0^1 \chi_0(z)dz = r\left(\int_0^{\delta/2} \chi_0(z)dz + \int_{\delta/2}^1 \chi_0(z)dz\right)$ becomes negative as $\delta \to 0$ and positive as $\delta \to 1$, the latter following from the analogous result in (i). Combining with $\frac{\partial}{\partial\delta}\chi_0(z) = \frac{\partial}{\partial\delta}\left(-\int_0^{q^p} xf(x)dx\right) = -q^p f(q^p)\frac{\partial}{\partial\delta}q^p > 0$, we conclude that $\pi_m^d - \pi_m^p$ crosses zero exactly once from negative to positive as δ changes from 0 to 1.

■

Proof of Lemma 3.

(i) It is clear that the analysis of the quantity precommitment and the price-quantity precommitment strategies are unchanged. As we found in Proposition 2, the latter strictly dominates the former, and the manufacturer's expected profit at the equilibrium under that strategy is $\pi_1 = r\mu\left(F^{-1}\left(1 - \frac{1}{r}\left[\gamma_1 + \frac{\delta\Delta_0}{2}\right]\right)\right)$. Now consider price precommitment. Since there is no demand uncertainty at the time quantity is decided, the manufacturer simply sets $q = D$. Anticipating this, the supplier chooses θ that maximizes his expected profit $\int_0^1 (w - G^{-1}(z \,|\, \theta))E[q]dz = \left(w - \int_0^1 G^{-1}(z \,|\, \theta)dz\right)\mu$, which is at $\theta = 1$. Anticipating this, the manufacturer then chooses w to maximize his expected profit $E[r\min\{D, q\} - wq] = (r - w)\mu$ subject to the participation constraint $(w - G^{-1}(z \,|\, 1))q \geq 0$, $\forall z \in [0, 1]$. The solution is $w = G^{-1}(1 \,|\, 1) = \gamma_1 + \delta\Delta_0/2$. Hence, the manufacturer's expected profit at the equilibrium is $\pi_2 = \left(r - \gamma_1 - \frac{\delta\Delta_0}{2}\right)\mu$. To compare π_1 and π_2, let $y \equiv F^{-1}\left(1 - \frac{1}{r}\left[\gamma_1 + \frac{\delta\Delta_0}{2}\right]\right)$. Then $\pi_1 = r\mu\,(y)$ and $\pi_2 = rF(y)\mu$. Define $\beta(y) \equiv F(y)\mu - \mu\,(y) = F(y)\mu - \int_0^y xf(x)dx$. Observe that $\beta(0) = 0$, $\lim_{y\to\infty} \beta(y) = 0$, and $\beta'(y) = (\mu - y)f(y)$. Since $\beta(y)$ starts from zero, increases in $0 \leq y < \mu$, and decreases in $\mu < y < \infty$ until it approaches zero again, $\beta(y) > 0$ for $y > 0$. Since $F^{-1}\left(1 - \frac{1}{r}\left[\gamma_1 + \frac{\delta\Delta_0}{2}\right]\right) > 0$, we conclude that $\pi_2 - \pi_1 = r\beta(y) > 0$. Therefore, the manufacturer's expected profit under price commitment, $\pi_m^p = \pi_2 = \left(r - \gamma_1 - \frac{\delta\Delta_0}{2}\right)\mu$, is strictly greater than that under price-quantity commitment.

(ii) Under the delay strategy, the manufacturer may offer a menu of contracts $\{(w(z \mid \theta), q(z \mid \theta))\}$ after demand is realized, which is after the supplier chooses the collaboration level θ in the product design stage. The manufacturer's problem is similar to $(\mathcal{D}_m)$ in Section 5.2 except that her objective function is free of expectation, i.e., $\int_0^1 (r \min\{D, q(z \mid \theta)\} - w(z \mid \theta) q(z \mid \theta))\, dz$. Proceeding the analysis similarly to the proof of Lemma 2, we can show that the problem reduces to

$$\max_{q(z \mid \theta)} \int_0^1 \left(r \min\{D, q(z \mid \theta)\} - [G^{-1}(z \mid \theta) + \nu(z \mid \theta)] q(z \mid \theta)\right) dz,$$

where $\nu(z \mid \theta) = \Delta_0 (1 - (1-\delta)\theta)\, z$. Note that $G^{-1}(z \mid \theta) + \nu(z \mid \theta) \le (\gamma_0 + \Delta_0/2) + \Delta_0 = \gamma_0 + (3/2)\Delta_0 < r$, according to the assumption in Section 3.1. Differentiating the objective with respect to $q(z \mid \theta)$, the derivative of the integrand of the objective function is $r - G^{-1}(z \mid \theta) - \nu(z \mid \theta) > 0$ if $q(z \mid \theta) < D$ and $-[G^{-1}(z \mid \theta) + \nu(z \mid \theta)] < 0$ if $q(z \mid \theta) > D$. In other words, the objective increases in $q(z \mid \theta)$ until $q(z \mid \theta) = D$ and then decreases. Therefore, it is optimal to choose $q^*(z \mid \theta) = D$ regardless of z or θ. Substituting this back into the objective and taking expectation, the manufacturer's expected profit becomes $\pi_m^*(\theta) = \mu \int_0^1 \left(r - G^{-1}(z \mid \theta) - \nu(z \mid \theta)\right) dz = \mu \left(r - \gamma_0 + (\gamma_0 - \gamma_1)\theta - \frac{1}{2}\Delta_0 (1 - (1-\delta)\theta)\right)$. The supplier's expected profit is (also found from the proof of Lemma 2) $\pi_s^*(\theta) = \int_0^1 \nu(z \mid \theta) E[q^*(z \mid \theta)] dz = \mu \int_0^1 \nu(z \mid \theta) dz = \frac{\mu}{2}\Delta_0 (1 - (1-\delta)\theta)$, which decreases in θ. Hence, the supplier chooses $\theta = 0$. Then the manufacturer's expected profit at the equilibrium is $\pi_m^d = \pi_m^*(0) = \left(r - \gamma_0 - \frac{\Delta_0}{2}\right)\mu$.

■

Proof of Proposition 7. It is easy to show that the integrated firm's expected profits under precommitment and under the delay strategy are, respectively, $\Pi_p^I = r\mu \left(F^{-1}\left(1 - \frac{\gamma_1}{r}\right)\right)$ and $\Pi_d^I = (r - \gamma_1)\mu$. It is optimal to choose $\theta = 1$ under both strategies. The comparison between Π_p^I and Π_d^I is similar to the analysis found in the proof of Lemma 7, part (i), showing that $\Pi_p^I < \Pi_d^I$. Now consider a decentralized supply chain. From the proof of Lemma 3, the manufacturer's expected profits at the equilibrium under price commitment and under the delay strategy are, respectively, $\pi_m^p = \left(r - \gamma_1 - \frac{\delta\Delta_0}{2}\right)\mu$ and $\pi_m^d = \left(r - \gamma_0 - \frac{\Delta_0}{2}\right)\mu$. Since $\gamma_0 > \gamma_1$ and $\delta < 1$ (from Assumption 2), $\pi_m^p > \pi_m^d$. ■