

Suicide Bombings and Targeted Killings in (Counter-) Terror Games

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This article develops sequential game models for key operational terrorist (how often to attack) and government (how often to execute targeted killings) decisions taken during a (counter-) terror campaign such as the second intifada. Key results include the following: The government initiates targeted killings when the marginal number of Israeli civilian lives saved from prevented terror attacks exceeds the marginal number of Palestinian civilian lives lost in such “hits”; targeted killings are not employed if they are either ineffective or extremely effective at thwarting terror (since terrorists will not induce their use); even after accounting for downstream terror attacks motivated by prior targeted killings, a civilian casualty-minimizing government can optimally order targeted killings over time; and low-level cycles of violence can occur when the government is more patient than the terrorists, but higher, stable levels of violence result when the terrorists are the more patient players in the game.

Keywords: *suicide bombings; targeted killings; game theory; operational decision making; terrorism; counterterrorism; Israeli–Palestinian conflict*

The second intifada pitting Palestinian terrorists against the State of Israel witnessed waves of suicide bombing attacks on Israeli civilians and targeted killings of terrorist operatives and leaders by the Israeli military that also resulted in Palestinian civilian casualties. This article models that conflict as an alternating sequence of actions taken by the terrorists and the government, with the optimal strategies given each side’s motivations determined from a game-theoretic perspective. In particular, we focus on the terrorists’ use of suicide bombing attacks and the government’s targeted killings of suspected terrorists by way of helicopter gunfire, fighter aircraft, and other means.

Israeli decisions to order targeted killings (or “hits”) are not taken lightly. The judge advocate general of the Israel Defense Forces (IDF) has listed the conditions

under which hits can be executed (Harel and Alon 2002; David 2003; Guiora 2004). There must be substantial evidence that the terrorists targeted are planning or will carry out attacks in the near future, thus hits are not intended as retribution for past events. The conditions also state that hits may occur only after appeals to the Palestinian Authority for arrests of those targeted have been ignored, and attempts by the IDF to arrest the suspects have failed or proven impossible. While several high-ranking leaders have been killed via hits since 2000, the IDF has focused largely on midlevel terrorists who are believed to be organizing and planning bombing attacks for the near future (David 2003).

The process that leads to a targeted killing generally begins with intelligence agencies collecting information on a suspect's past and expected future activity, often through the testimony of collaborators. The information is then detailed in a report given to senior IDF officials, who make a recommendation to the IDF chief of staff regarding whether to target the suspect. The chief of staff must gain approval from the Israeli cabinet, and if the risk of Palestinian civilian casualties is high, he often seeks the final approval of the minister of defense and the prime minister. This thorough process leading to the approval of targeted killings sets hits apart from other counterterrorism measures (Harel and Alon 2002; David 2003; Guiora 2004; Izenberg 2006; Katz 2006).

Many critics have debated the legality of targeted killings under international law, while others question the morality of pursuing a policy that risks the lives of civilian bystanders (David 2003, Dershowitz 2004, Gross 2003, Guiora 2004; Katz 2006). Others have suggested that targeted killings promote terrorism via the recruitment of revenge-seeking individuals to terror organizations (Atran 2003; Ganor 2005; Pape 2003), and statistical investigations of the relationship between targeted killings and downstream suicide bombing attacks in the Israeli–Palestinian conflict appear to confirm this view (Kaplan et al. 2005; Kaplan, Mintz, and Mishal 2006). This article does not attempt to address the legal or ethical questions surrounding targeted killings. Rather, we ask whether hits can be an optimal counterterror measure when evaluated in terms of saving total (Palestinian and Israeli) civilian lives, even when hits provide downstream benefits to terrorists because of recruitment.

As for the terrorists, some question whether they behave rationally (Abrahms 2004, 2007). To gain some insight into one arch-terrorist's mind-set, consider the words of Muhammad Deif, the commander of the Hamas military wing (as cited in Al-Hajjar 2004): "Believe me, entire families, in some cases, wish to volunteer in carrying out 'martyrdom operations.' However, the large number of those 'living martyrs' cannot blind us from the fact that their blood is dear, and we are not prepared to waste it in cases in which there is a possibility that an attacker would not be able to reach his target and carry out a successful attack." This article analyzes terrorist decision making that seeks, as Deif implies, to maximize the success of terror attacks while considering their attendant costs.

Game theory has proven valuable in the study of terrorism and counterterrorism. As stated by Arce and Sandler (2005, p. 184), "Game theory is an appropriate tool for investigating counterterrorism because it captures the strategic interactions between terrorists and targeted governments whose choices are interdependent." Some recent studies applying game theory to various aspects of terrorism and counterterrorism include Arce and Sandler (2005), Brown et al. (2006), Bueno de Mesquita (2005a, 2005b), Heal and Kunreuther (2005), Siqueira and Sandler (2006), and Wein and Baveja (2005). For a detailed review, see Sandler and Arce (2007); also see volume 49, number 2, of the *Journal of Conflict Resolution* for additional articles.

With the exception of the infrastructure defense models of Brown et al. (2006) and Wein and Baveja's (2005) analysis of optimal fingerprinting in the U.S. Visitor and Immigrant Status Indicator Technology Program, the articles cited earlier focus on broad strategic questions, such as the relative attractiveness of deterrence versus preemption in counterterrorism (Arce and Sandler 2005) or the competition between terrorists and governments to win the support of the civilian population (Siqueira and Sandler 2006). By contrast, the present article models the key operational decisions faced by Palestinian terrorists (how often to dispatch suicide bombers) and the Israeli government (how often to conduct targeted killings) during the second intifada while endogenizing Israeli preparedness for terror attacks as a function of the planned attack rate.

In the next section, we introduce a two-period game played between the terrorists and the government. The game features terrorist recruitment in period 2 in response to government hits in period 1. The game can lead to scenarios where the level of violence (as characterized by the rates of suicide bombing attacks and targeted killings) escalates or de-escalates from the first period to the second. In section 2, we generalize this model to the multiperiod setting. This dynamic model is of special interest, for to quote from Sandler and Arce (2007, p. 44, in preprint at <http://tinyurl.com/y2x588>), "there is currently no multiperiod analysis of terrorist campaigns where terrorists must choose the patterns of attacks for three or more periods." Our analysis of the multiperiod model proceeds via dynamic programming. We allow the terrorists and the government to discount the future at different rates, which leads naturally to the role of patience in (counter-) terrorism games. We find that when the terrorists are the more patient players in the game, the levels of violence tend toward a stationary equilibrium. Conversely, when the government is more patient than the terrorists, cyclic and even chaotic fluctuations in the suicide bombing and hit rates can occur over time. We illustrate the models with data derived from the peak years of the second intifada (2001 to 2003).

A Two-Period Game

We begin with a two-period sequential game between terrorists and the government. Each period has two stages. The terrorists move first by deciding how many

suicide bombings to attempt in the period. The government moves second by deciding how many targeted killings (or hits) to order with the hope of preempting planned suicide bombings. Payoffs occur at the end of each period, with the terrorists maximizing the difference between the benefits and costs of planned suicide bombing attacks while the government maximizes the net benefits of targeted killings. In the second period, terrorists also derive recruiting benefits depending on government actions in the first period.

We assume perfect information: the government, via intelligence and surveillance measures, knows the terrorists' planned attack rates, while the terrorists in turn know that the government can divine their plans. While this assumption is common in game-theory models of this type, it is perhaps more appropriate in this application than in many other settings given Israel's extensive intelligence-gathering capabilities.

Let λ_t denote the period t suicide bombing attack rate, which is the terrorists' planned number of attacks in the period. Absent any government attempt to preempt these planned suicide bombings, attacks can still fail. The probability an attack fails depends on the government's state of preparedness, which in turn depends (via intelligence/perfect information) on the terrorists' planned attack rate. Consider the government's response to increased threats of terrorism: as the expected attack rate increases, so does the government's state of alertness. This heightened alertness leads to increased arrests and heightened vigilance in other security measures, such as checkpoints, roadblocks, and general surveillance, decreasing the chance of a successful attack. To reflect these relationships between planned terror attacks and defensive preparedness, we assume that the probability a planned attack succeeds when the terrorists have planned λ attacks equals the non-increasing function $\pi(\lambda)$. One may question why we treat these defensive security measures differently than the strategic decision making involved in ordering hits to be described below. While there is certainly some interplay between hits and defensive measures, we do not examine this relationship for the purpose of this analysis. Thus, absent preemptive actions by the government, the expected number of suicide bombings that succeed in a period when λ such attacks are planned is given by $\lambda\pi(\lambda)$.

In the second stage of period t , the government strategically chooses the number of hits to undertake with the goal of preempting planned attacks. Consistent with the IDF's criterion that hits can be used only to prevent future attacks (David 2003; Harel and Alon 2002; Izenberg 2006), we assume that in period t , the government targets only individuals who are integral in planning and organizing attacks planned for period t . Let x_t denote the number of hits ordered by the government (referred to as the *hit rate*) in period t .

It is likely that targeted terrorists are responsible for planning only some of the attacks to take place in the period, so while a hit has a high probability of thwarting these attacks, the chance of thwarting others is much lower. Even if the targeted

terrorists are killed, it could be that planning for these attacks had progressed to the point that they could be carried out without further guidance. It is also possible for a hit to go awry and miss the terrorists targeted altogether (Kaplan et al. 2005). Taking these considerations into account, we assume that if x hits are ordered, the number of successful suicide bombing attacks when λ such attacks have been planned is reduced from $\lambda\pi(\lambda)$ to $\lambda\pi(\lambda)\exp(-rx)$ (thus each hit reduces the number of successful attacks by the multiplicative factor $\exp(-r)$).

Terrorists derive benefits in proportion to the number of successful suicide bombing attacks. We denote this benefit by s . In our analysis and examples below, we consider s to be the expected number of Israeli civilians killed per successful suicide bombing attack, though we realize that the terror benefits of an attack could extend beyond the murder of civilians to include the number wounded, property damage, or the broader instillation of fear in the population. When the terrorists plan λ attacks and the government executes x hits in a period in this game, the benefit terrorists derive from suicide bombings is given by $\lambda\pi(\lambda)\exp(-rx)s$, while the government suffers losses of the same amount.

Many have argued that targeted killings or other offensive or retaliatory counter-terror measures contribute to the recruitment of new terrorists (Atran 2003; Brophy-Baermann and Conybeare 1994; Enders, Sandler, and Cauley 1990; Enders and Sandler 1993; Ganor 2005; Kaplan et al. 2005; Pape 2003). Rosendorff and Sandler (2004) present a one-period model with terrorist recruitment in which the government first chooses a proactive response level after which the terrorists decide whether to attempt a small- or large-scale attack. In that model, the level of terrorist recruitment is proportional to the government's proactive response level and is higher with a successful large-scale attack.

In our model, we assume "hit-dependent" recruitment. Hits create a strong desire for terrorist retaliation. For every hit carried out by the government in period 1, such desire creates a pool of potential recruits in period 2 who wish to see immediate action taken against the government. However, the number of recruits the terrorists receive from this pool depends on the number of terror attacks planned in period 2: if more retaliatory attacks are planned, more recruits join the terrorists, while if few attacks are planned for period 2, the terrorists will lose many potential recruits who wanted to see action taken immediately. Note that new recruits need not be the ones to carry out suicide bombings in period 2, but they are viewed as additional terrorist resources. Indeed, the number of recruits in period 2 could easily exceed the number of bombings planned for that period as suggested by the terrorism literature. As the planned period 2 attack rate increases, a growing percentage of the recruitment pool joins the terrorists, leaving a smaller percentage of the pool still available for further increases in the attack rate. Thus, in our model, actual recruitment exhibits diminishing marginal returns in the period 2 attack rate. We capture the terrorists' benefits of hit-dependent recruitment by introducing a recruitment term to the terrorists' period 2 payoff. We model such recruitment

benefits as $\beta x_1 \lambda_2 / (x_1 + \lambda_2)$, where β represents the number of potential recruits generated per targeted hit in period 1, βx_1 is thus the total pool of potential recruits following x_1 hits in period 1, and $\lambda_2 / (x_1 + \lambda_2)$ is the fraction of the pool that actually joins the terrorists if λ_2 attacks are planned in period 2.

Finally, both the terrorists and the government face costs associated with their choices. The terrorists face costs in the planning and execution of each suicide bombing attack that include the time and resources needed to organize attacks and the lives of the terrorists who die or are captured in attempting them. Let α denote the cost per attack planned. Similarly, let c denote the expected cost per hit to the government. One way to think about the cost of hits is to focus on Palestinian civilian casualties that result from targeted killings. While some might argue that the government does not take civilian casualties into account, one must consider that otherwise, the government would simply hit as many times as their resources allowed. And while the government does consume resources such as missiles, fuel, planes, and pilots in performing hits, in this analysis we focus on the human costs and benefits of the government's decision.

Estimating the Evasion Probability $\pi(\lambda)$

We model the evasion probability $\pi(\lambda)$ using historical data from 2001 to 2003, when violence in the conflict was at its peak (Kaplan et al. 2005; Kaplan, Mintz, and Mishal 2006). The fraction of planned attacks that succeeded in a given month is estimated as

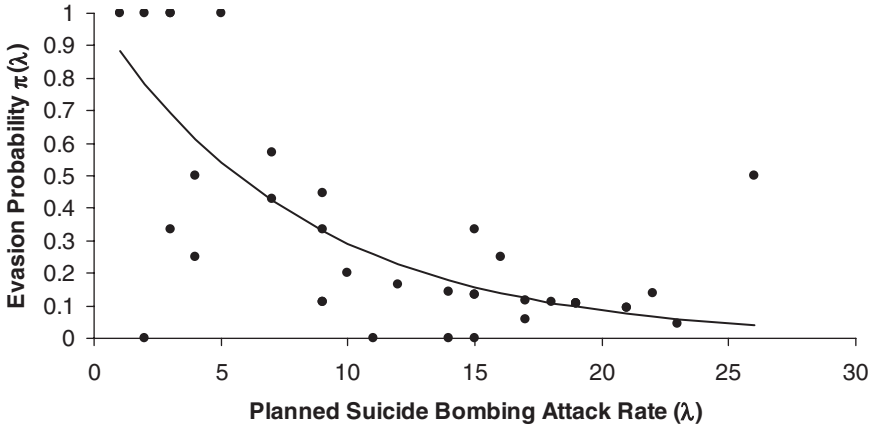
$$\pi(\lambda) = \frac{\text{\# of suicide bombings}}{\text{\# of arrests} + \text{\# of interceptions} + \text{\# of suicide bombings}}, \quad (1)$$

where arrests are of persons suspected of planning suicide bombings, and interceptions refer to terrorists interdicted en route to their targets (Kaplan et al. 2005; Kaplan, Mintz, and Mishal 2006). The denominator in equation (1) thus estimates the total number of suicide bombing attacks λ planned for a given month. This assumes that each arrest and interception would have resulted in a suicide attack. While this assumption is not entirely accurate, the sum can be considered proportional to the total number of planned attacks, thus changes in the given fraction from period to period do reflect $\pi(\lambda)$ as a function of λ . Figure 1 plots equation (1) versus the estimated total number of planned suicide bombings for each month during 2001 to 2003. In light of Figure 1, we assume that

$$\pi(\lambda) = e^{-\mu\lambda} \quad (2)$$

for $\mu > 0$, from which we obtain the estimate $\mu = 0.123$ and hence $\pi(\lambda) = \exp(-0.123\lambda)$. In the interest of simplicity and in view of Figure 1, we will assume that $\pi(\lambda)$ follows equation (2) for the duration of this article.

Figure 1
Attack Rate–dependent Evasion Probabilities:
Observed (equation [1], dots) and Modeled ($e^{-0.123\lambda}$, curve)



Payoffs

Let $U_t^G(\lambda_t, x_t)$ denote the period t payoff to the government. These payoffs are given by

$$U_t^G(\lambda_t, x_t) = -\lambda_t e^{-\mu\lambda_t} e^{-rx_t} s - cx_t \quad t = 1, 2. \tag{3}$$

The payoff to the government over the entire game is just the sum of the period-specific payoffs. If one interprets s and c as the expected number of Israeli civilians killed per suicide bombing and the expected number of Palestinian civilians killed per hit, respectively, maximizing the government’s payoff over the entire game is equivalent to minimizing total civilian deaths.

Let $U_1^T(\lambda_1, x_1)$ denote the first period payoff to the terrorists, which is given by

$$U_1^T(\lambda_1, x_1) = \lambda_1 e^{-\mu\lambda_1} e^{-rx_1} s - \alpha\lambda_1. \tag{4}$$

The terrorists’ payoff in the second period includes the recruitment benefit and equals

$$U_2^T(\lambda_2, x_1, x_2) = \lambda_2 e^{-\mu\lambda_2} e^{-rx_2} s - \alpha\lambda_2 + \beta x_1 \frac{\lambda_2}{x_1 + \lambda_2}. \tag{5}$$

The terrorists’ payoff for the full game is just the sum of their period-specific payoffs.

Equilibrium

We obtain the subgame perfect equilibrium via backward induction. In the second period, the government chooses its hit rate x_2 by maximizing $U_2^G(\lambda_2, x_2)$ subject to $x_2 \geq 0$. We solve for x_2 by differentiating $U_2^G(\lambda_2, x_2)$ with respect to x_2 and setting the result to zero, which leads to

$$\lambda_2 e^{-\mu\lambda_2} e^{-rx_2} s = \frac{c}{r}. \quad (6)$$

Using equation (6), the best response function for x_2 is solved as a function of λ_2 , subject to the constraint that $x_2 \geq 0$. The result is the optimal hit rate function $x_2^*(\lambda_2)$ given by

$$x_2^*(\lambda_2) = \max\left(\frac{1}{r} \log\left(\frac{\lambda_2 e^{-\mu\lambda_2} rs}{c}\right), 0\right). \quad (7)$$

Note that it can be optimal for the government to “sit,” that is, not to order any targeted killings. Note that the government hits only when $\lambda_2 e^{-\mu\lambda_2} rs > c$, that is, when the marginal benefit of initiating hits exceeds the marginal cost of doing so. When s and c are denominated in terms of Israeli and Palestinian civilian lives, this says that the government hits only when the marginal number of Israeli civilians saved from suicide bombings averted by initiating hits exceeds the marginal number of Palestinian civilians killed by initiating hits.

Given $x_2^*(\lambda_2)$ as well as the period 1 hit rate x_1 , the terrorists choose the period 2 attack rate $\lambda_2^*(x_1)$ that maximizes $U_2^T(\lambda_2, x_1, x_2^*(\lambda_2))$. Hence, $\lambda_2^*(x_1)$ is the solution to

$$\max_{\lambda_2 \geq 0} U_2^T(\lambda_2, x_1, x_2^*(\lambda_2)) = \lambda_2 e^{-\mu\lambda_2} e^{-rx_2^*(\lambda_2)} s - \alpha\lambda_2 + \beta x_1 \frac{\lambda_2}{x_1 + \lambda_2}. \quad (8)$$

Let $\lambda_2'(x_1)$ maximize the terrorists' second period utility given that the government sits in the second period, that is,

$$\lambda_2'(x_1) = \arg \max_{\lambda \geq 0} \lambda e^{-\mu\lambda} s - \alpha\lambda + \beta x_1 \frac{\lambda}{x_1 + \lambda}. \quad (9)$$

At this attack rate, the government would sit if its marginal benefit from initiating hits ($\lambda_2'(x_1) \exp(-\mu\lambda_2'(x_1))rs$) is smaller than the marginal cost of doing so (c). If this is the case, then the terrorists will set $\lambda_2(x_1) = \lambda_2'(x_1)$ as they can be assured that the government will in fact sit.

However, if the government's marginal benefit of initiating hits at $\lambda_2'(x_1)$ exceeds its marginal cost, that is, if $\lambda_2'(x_1) \exp(-\mu\lambda_2'(x_1))rs > c$, then define λ^+ as the smallest root of the equation

$$\lambda e^{-\mu\lambda} rs = c. \quad (10)$$

Equation (10) equalizes the government's marginal cost and benefit of initiating hits. λ^+ is thus the largest attack rate the government will tolerate before initiating

hits. If the terrorists plan $\lambda_2 > \lambda^+$ attacks in period 2, the government hits in accord with equation (7), which fixes the expected number of Israeli civilian casualties in the second period to equal c/r (see equation [6]), and the terrorists seek to maximize

$$U_2^T(\lambda_2, x_1, x_2^*(\lambda_2)) = c/r - \alpha\lambda_2 + \beta x_1 \frac{\lambda_2}{x_1 + \lambda_2}. \tag{11}$$

Differentiating the expression above with respect to λ_2 and equating to zero leads to the function

$$\lambda_2''(x_1) = (\sqrt{\beta/\alpha} - 1)x_1. \tag{12}$$

The function $\lambda_2''(x_1)$ is the optimal period 2 terror attack rate providing that the government hits in period 2, which it will if $\lambda_2''(x_1)$ exceeds the hit-inducing threshold λ^+ . If $\lambda_2''(x_1) \leq \lambda^+$, however, then the terrorists will attack at the threshold rate λ^+ without inducing a hit.

To summarize, the equilibrium terror attack rate in period 2 as a function of the government’s hit rate in period 1 is given by

$$\lambda_2^*(x_1) = \begin{cases} \lambda_2'(x_1) & \text{if } \lambda_2'(x_1) \exp(-\mu\lambda_2'(x_1))rs \leq c \\ \max(\lambda_2''(x_1), \lambda^+) & \text{if } \lambda_2'(x_1) \exp(-\mu\lambda_2'(x_1))rs > c. \end{cases} \tag{13}$$

Equation (13) shows how the terrorists’ optimal play in period 2 depends on the government’s hit rate in period 1. The terrorists first see if they can achieve optimality without inducing the government to hit in the second period. If so, that is what they do by planning $\lambda_2'(x_1)$ attacks in period 2. If not, then the terrorists consider the best result they can achieve without inducing a hit, which is to attack at the threshold rate λ^+ , and see if they can do better by inducing the government to hit. When inducing the government to hit proves optimal, the terrorists balance the benefits of recruitment versus the costs of period 2 terror attacks (hence the ratio of β/α in equation [12]) and conclude that the number of attacks planned in period 2 should be proportional to the number of hits in period 1.

Continuing with backward induction, the government chooses its first period hit rate x_1 to maximize its total game utility, with the knowledge that x_1 dictates the terrorists’ period 2 attack rate λ_2 , and that λ_2 subsequently dictates the government’s period 2 hit rate x_2 . The government does this conditional upon λ_1 , the terrorists’ attack rate in period 1. The government therefore determines the function $x_1^*(\lambda_1)$ as

$$\begin{aligned} x_1^*(\lambda_1) &= \arg \max_{x_1 \geq 0} \{U_1^G(\lambda_1, x_1) + U_2^G(\lambda_2^*(x_1), x_2^*(\lambda_2^*(x_1)))\} \\ &= \arg \max_{x_1 \geq 0} \left\{ -\lambda_1 e^{-\mu\lambda_1} e^{-rx_1} s - cx_1 \right. \\ &\quad \left. - \lambda_2^*(x_1) e^{-\mu\lambda_2^*(x_1)} e^{-rx_2^*(\lambda_2^*(x_1))} s - cx_2^*(\lambda_2^*(x_1)) \right\}, \end{aligned} \tag{14}$$

Table 1
Parameter Values Used in Numerical Examples

Parameter	Symbol	Numerical Value
Israeli civilian deaths per suicide bombing	s	5.5
Palestinian civilian deaths per targeted killing	c	1
Terrorist cost per planned suicide bombing	α	1
Terrorist evasion probability decay rate	μ	0.123
Terrorist recruitment pool generated per hit	β	6.8

where the functions $\lambda_2^*(\cdot)$ and $x_2^*(\cdot)$ are found from equations (13) and (7), respectively.

The terrorists then choose the attack rate λ_1^* that maximizes their total utility, anticipating the downstream consequences of this choice. Given the functions $x_1^*(\cdot)$, $\lambda_2^*(\cdot)$, and $x_2^*(\cdot)$ derived above, the terrorists select their attack rate in period 1 as

$$\lambda_1^* = \arg \max_{\lambda \geq 0} \left\{ \begin{array}{l} \lambda e^{-\mu\lambda} e^{-r x_1^*(\lambda)} s - \alpha \lambda \\ + \lambda_2^*(x_1^*(\lambda)) e^{-\mu\lambda_2^*(x_1^*(\lambda))} e^{-r x_2^*(\lambda_2^*(x_1^*(\lambda)))} s - \alpha \lambda_2^*(x_1^*(\lambda)) \\ + \beta x_1^*(\lambda) \frac{\lambda_2^*(x_1^*(\lambda))}{x_1^*(\lambda) + \lambda_2^*(x_1^*(\lambda))} \end{array} \right\}. \quad (15)$$

The optimizations in equations (14) to (15) must be solved numerically, but different structural equilibria emerge, as discussed next.

Numerical Examples and Structural Equilibria

We further draw on the data employed by Kaplan et al. (2005; Kaplan, Mintz, and Mishal 2006) to set additional parameters (Table 1). During 2001 to 2003, there were 75 targeted killings and 80 civilian casualties resulting from those hits, indicating that the number of Palestinian civilians killed per targeted hit $c \approx 1$. During these same months, there were 471 Israeli civilians killed in 85 suicide bombing attacks within the 1967 Green Line, resulting in an estimate of $s = 5.5$. With respect to the cost of a suicide bombing attack, although terrorists do face resource costs, we continue to focus on human costs and assume that $\alpha = 1$ because one terrorist dies or is captured per actual or interdicted suicide bombing. We add the additional assumption that β , the number of potential terror recruits per hit, equals 6.8. This value is taken from the study of Kaplan et al. (2005), who estimated that Israeli hits generated 8.6 new recruits while killing 1.8 terror suspects per hit (as distinct from civilian deaths), yielding net recruitment of 6.8. The remaining unknown parameter is the hit effectiveness r , whose estimation is beyond the scope of this article. We therefore solve equations (7), (13), (14), and (15) parametrically in r .

Figure 2a
Structural Equilibria in Two-period Games: Escalation

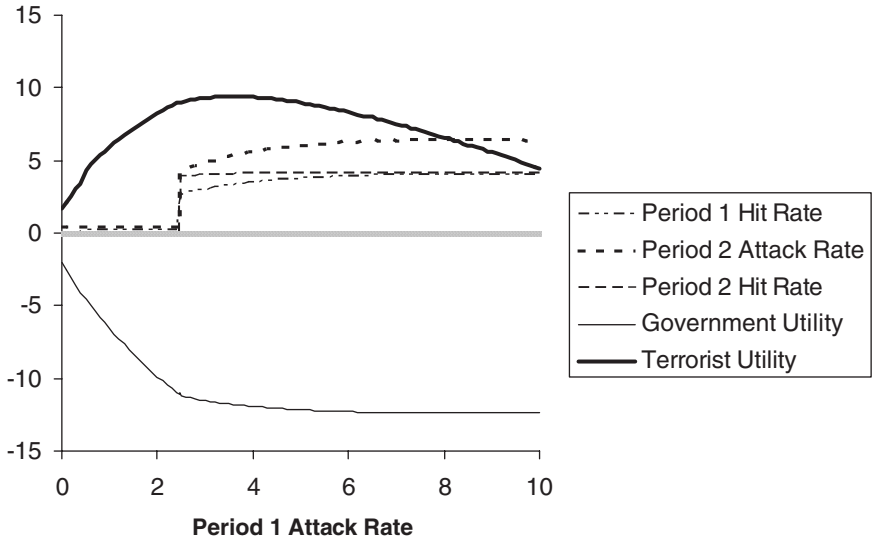


Figure 2a reports the complete equilibrium solution to the game when $r = 0.5$. The graph plots the period 1 and 2 hit rates, the period 2 attack rate, and overall payoffs to the terrorists and the government as a function of the period 1 attack rate. In each case, the equilibrium is found by locating the period 1 attack rate that maximizes overall payoffs to the terrorists and then reading off the associated values of the other functions.

When $r = 0.5$, hits prevent attacks with probability $1 - \exp(-.5) \approx 0.4$. The total terror payoffs are maximized at $\lambda_1^* = 3.43$, which induces hits at rate $x_1^* = 3.34$ and $x_2^* = 4.05$, as well as second-period attacks at rate $\lambda_2^* = 5.3$. As both the attack rates and hit rates increase from period 1 to period 2, we call this an *escalation* equilibrium. For the parameter values in Table 1, escalation equilibria feature when $r \geq 0.3$, though as r becomes very large and hits are essentially guaranteed to thwart suicide bombings, the associated attack and hit rates tend to zero. For example, at $r = 10$ (which prevents attacks with probability $1 - \exp(-10) \approx 0.99995$), $\lambda_1^* = 0.3$, $\lambda_2^* = 0.4$, $x_1^* = 0.26$, and $x_2^* = 0.3$.

Figure 2a reveals more about the structure of optimal play in this game. Note that until the period 1 attack rate λ_1 reaches 2.46, the government hits only at the low rate of 0.24 in the first period. This hit rate is special because it induces the terrorists

to attack in the second period with rate $\lambda_2 = (\sqrt{\beta/\alpha} - 1) \times x_1 = 1.61 \times 0.24 = 0.38$, which coincides with the second-period hit triggering threshold λ^+ . This means that the government will not hit in the second period. This example provides a vivid illustration of the decision facing the government in the first period: it can either hit at a low rate that ensures the terrorists will attack only at the threshold rate in the second period, allowing the government to sit; or it can hit at a higher rate that will lead to a sufficiently high rate of second-period attacks to force the government to also hit in the second period. The government can withstand up to 2.46 attacks in the first period without responding in a major way, but once the attack rate exceeds that level, the government must hit back to save more lives. The terrorists realize that this is the case and select their first-period attack rate accordingly. While the terrorists force the government to escalate in this example's equilibrium solution, that the government exhibits restraint until the terrorists plan more than 2.46 first-period attacks reveals how the prospect of recruitment-motivated attacks in the second period can compel the government to display patience in the first period.

There are other equilibria that result for different values of r . For example, when $0 \leq r < 0.065$, hits are so ineffective that they are never employed by the government, which in turn deprives the terrorists of potential recruitment benefits. We call this a *stationary sit* equilibrium; the terrorists attack at the optimal rate $\lambda' = 5.29$ in both periods while the government sits. Figure 2b reports another example: when $r = 0.07$, the optimal period 1 attack rate solves to $\lambda_1^* = 6.94$, which in turn sets the equilibrium values for the other decision variables at $x_1^* = 1.85$, $\lambda_2^* = 4.54$, and $x_2^* = 0$. In this example, $\lambda_1^* > \lambda_2^*$ and $x_1^* > x_2^*$. We refer to this as a *de-escalation* equilibrium, because both the terror attack and government hit rates decline from the first period to the second. De-escalation equilibria are sustained for hit effectiveness values in the range $0.065 \leq r < 0.257$.

Figure 3 plots the equilibrium strategies for the parameter values of this example as a function of r . The different equilibrium structures are clearly evident from this graph. Hit-dependent recruitment can lead terrorists to attack at higher rates than they would have otherwise, with the hope of inducing the government to hit to generate new terror recruits.

But beyond these structural results, the model suggests something different about the use of aggressive counterterror tactics such as targeted killings: *hitting can be optimal for a civilian casualty-minimizing government, even when hits serve to recruit more terrorists*. This is particularly the case if hits are at least moderately effective. Prior research has emphasized that aggressive counterterror tactics can spark terror recruitment (Atran 2003; Brophy-Baermann and Conybeare 1994; Enders, Sandler, and Cauley 1990; Enders and Sandler 1993; Ganor 2005; Kaplan et al. 2005; Pape 2003). Rosendorff and Sandler (2004) also arrived at an equilibrium where an intermediate proactive response level was optimal for the government: its actions must be strong enough to hinder a large-scale terror attack, but small enough to deprive terrorists of sufficient recruits to accomplish a large-scale

Figure 2b
Structural Equilibria in Two-period Games: De-escalation

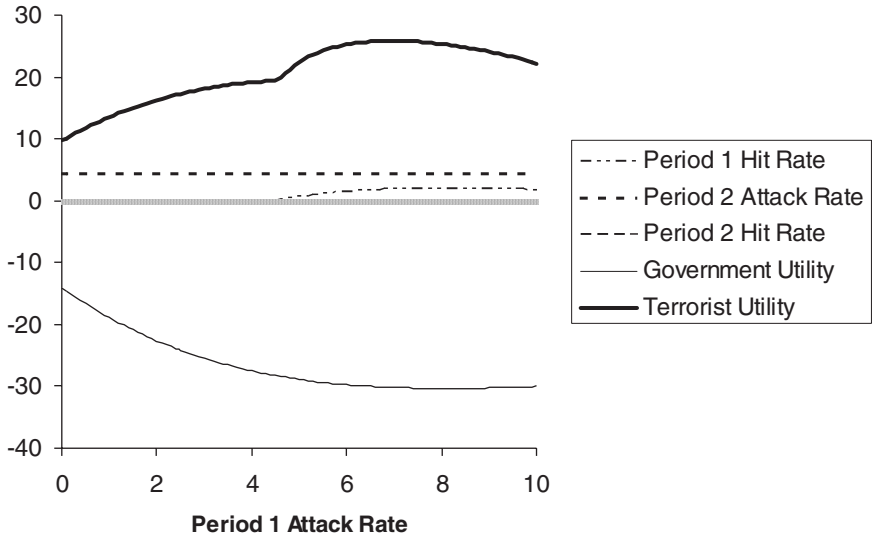
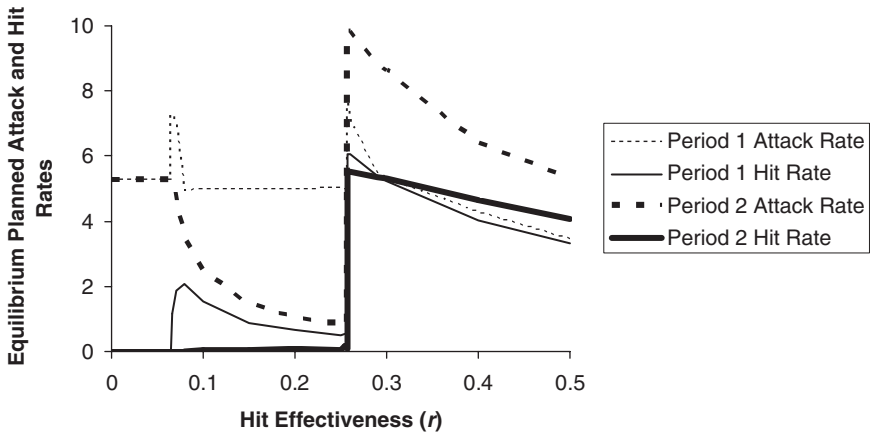


Figure 3
Parametric Solution to the Two-period Model in Hit Effectiveness r



attack. What is new in our results is the suggestion that hitting can be the optimal strategy for a rational government trying to minimize total civilian casualties on both sides, even with the knowledge that targeted killings lead to terror recruitment.

Discounted Repeated Games

We now extend our analysis to multiple periods. Across n periods, we retain the order of play: in period t , the terrorists plan attacks at rate λ_t while the government hits back with rate x_t , and hits in period t spark terrorist recruitment benefits in period $t + 1$. In addition, the terrorists and the government discount their future payoffs with discount factors θ^T and θ^G , respectively, per period. In formulation, this model somewhat resembles the alternating-sequence dynamic oligopoly models of Maskin and Tirole (1987, 1988).

We solve for the subgame perfect equilibria via dynamic programming. For notation, let λ_t and x_t denote arbitrary planned terror attack and government hit rates in period t . These are distinct from the *decision functions* $x_t^*(\lambda_t)$ and $\lambda_t^*(x_{t-1})$, which denote the *optimal* period t hit rate as a function of the period t terror attack rate, and the *optimal* period t attack rate as a function of the hit rate in period $t - 1$. These decision functions are derived from the *value functions*: the government's period t value function given the terrorists' planned attack rate λ_t is defined by

$$V_t^G(\lambda_t) = \max_{x_t \geq 0} \{ -\lambda_t e^{-\mu\lambda_t} e^{-rx_t} s - cx_t + \theta^G V_{t+1}^G(\lambda_{t+1}^*(x_t)) \} \text{ for } t = 1, 2, \dots, n, \tag{16}$$

from which the government's decision function $x_t^*(\lambda_t)$ is found as

$$x_t^*(\lambda_t) = \arg \max_{x_t \geq 0} \{ -\lambda_t e^{-\mu\lambda_t} e^{-rx_t} s - cx_t + \theta^G V_{t+1}^G(\lambda_{t+1}^*(x_t)) \} \text{ for } t = 1, 2, \dots, n. \tag{17}$$

We set $V_{n+1}^G(\cdot) = 0$, signaling the end of the conflict after n periods.

Similarly, we define the terrorists' period t value function given the government's hit rate from the prior period x_{t-1} as

$$V_t^T(x_{t-1}) = \max_{\lambda_t \geq 0} \left\{ \lambda_t e^{-\mu\lambda_t} e^{-rx_t^*(\lambda_t)} s - \alpha\lambda_t + \beta x_{t-1} \frac{\lambda_t}{x_{t-1} + \lambda_t} + \theta^T V_{t+1}^T(x_t^*(\lambda_t)) \right\} \text{ for } t = 2, 3, \dots, n \tag{18}$$

along with the first period optimal value

$$V_1^T = \max_{\lambda_1 \geq 0} \left\{ \lambda_1 e^{-\mu\lambda_1} e^{-rx_1^*(\lambda_1)} s - \alpha\lambda_1 + \theta^T V_2^T(x_1^*(\lambda_1)) \right\}. \tag{19}$$

The terrorists' decision function $\lambda_t^*(x_{t-1})$ is given by

$$\lambda_t^*(x_{t-1}) = \arg \max_{\lambda_t \geq 0} \left\{ \lambda_t e^{-\mu\lambda_t} e^{-rx_t^*(\lambda_t)} s - \alpha\lambda_t + \beta x_{t-1} \frac{\lambda_t}{x_{t-1} + \lambda_t} + \theta^T V_{t+1}^T(x_t^*(\lambda_t)) \right\} \text{ for } t = 2, 3, \dots, n, \tag{20}$$

while the optimal terror attack rate in the first period equals

$$\lambda_1^* = \arg \max_{\lambda_1 \geq 0} \left\{ \lambda_1 e^{-\mu\lambda_1} e^{-r x_1^*(\lambda_1)} s - \alpha \lambda_1 + \theta^T V_2^T(x_1^*(\lambda_1)) \right\}. \tag{21}$$

As with the government, we set $V_{n+1}^T(\cdot) = 0$ to close the game after n periods.

The discount factors θ^G and θ^T can be interpreted as measures of patience: the higher the discount factor, the more patient the player. Changing the discount factors while fixing all other parameter values leads to different qualitative results. While the exact dependence of the optimal strategies on the discount factors is complicated (not surprising, given equations [16] to [21]), numerical simulations reveal three main equilibrium behaviors: stationary sit, stationary hit, and (perhaps chaotic) cycles of violence.

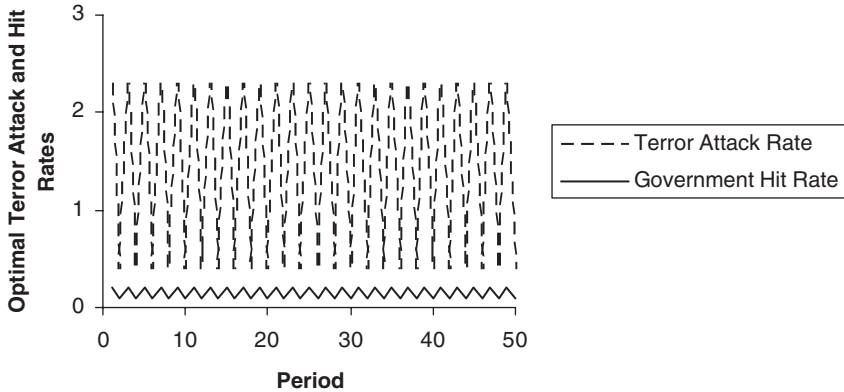
Stationary Sit Equilibrium

When θ^G and θ^T are both very small, play by either side in any period does not consider the consequences of present actions for future periods. This leads to repeated play of a single period game that results in a stationary sit equilibrium: the government never hits, and consequently the terrorists never gain recruitment benefits. In each period, the government sits while the terrorists attack at the rate that maximizes their per-period payoff (given by $\min(\lambda_0 2(0), \lambda +)$ from equations [9, 10]). For the parameter values of Table 1 and $r = 0.5$, we obtain this equilibrium for discount factors lower than 0.004.

Stationary Hit Equilibrium

A different and common equilibrium occurs when $\theta^G < \theta^T$ but θ^T is sufficiently large (at least 0.05 for the parameters in Table 1 if $r = 0.5$). After an initial transient period, the planned attack and hit rates converge to stable values λ^* and x^* , respectively, for the remainder of the game. The specific stationary values reached depend on the values of the discount factors (see Maskin and Tirole, 1987, for a direct approach to determining stationary values in a similar game). For example, in a game lasting $n = 50$ periods, the planned attack and hit rates converge to $\lambda^* \approx 6.94$ and $x^* \approx 4.13$. If $\theta^G = 0$ while $\theta^T = 1$, the government is reduced to a myopic period-by-period reactive role while the terrorists plan weighting each of the 50 periods equally. In this case, λ^* converges to about 7.24 attacks per period, while the government responds with $x^* \approx 4.2$. Intermediate discount factors yield intermediate results. In a 50-period game with $\theta^G = \theta^T = 0.5$, after an initial transient period, the planned attack and hit rates converge to $\lambda^* \approx 7$ and $x^* \approx 4.15$ for the remainder of the game.

Figure 4a
Discounted Repeated Games: Repeating Two-period Game ($\theta^G = 1, \theta^T = 0$)



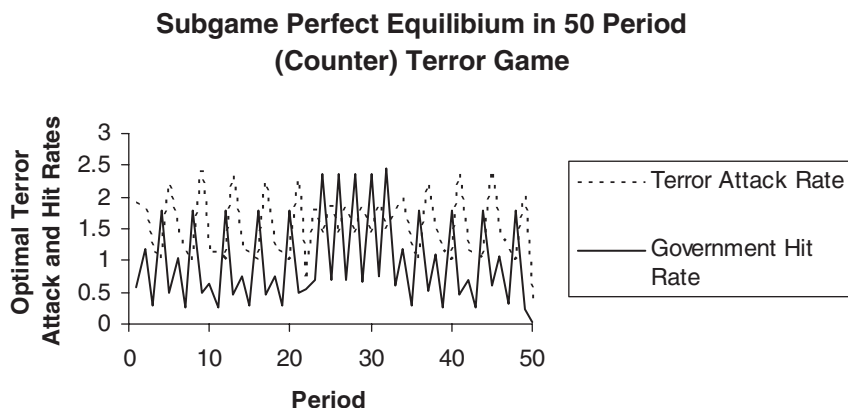
Cycles and Chaos

Quite different results occur when the government is the more patient player in the game. Figure 4a reports the equilibrium planned attack and hit rates in a 50-period game for the parameter values in Table 1 with $r = 0.5$ when $\theta^G = 1$ and $\theta^T = 0$. There is an obvious 2-period cycle: in odd-numbered periods, the planned attack rate $\lambda_{\text{odd}} = 2.46$, while in even-numbered periods, the planned attack rate $\lambda_{\text{even}} = 0.39$. Similarly, in odd-numbered periods the government hits with rate $x_{\text{odd}} = 0.24$, while in even-numbered periods the government sits.

What is causing this pattern? With $\theta^T = 0$, the terrorists do not take the future into account when planning attacks in any period, and in particular, they do not anticipate downstream recruitment benefits that will result from government hits. What is happening is continual repetition of the two-period game.

The government knows that hits drive recruitment; thus, the government does not want to hit too hard in the first period. In the present example, the government selects a hit rate $x = 0.24$ in the first period, knowing that in the second period the terrorists will set their attack rate equal to $(\sqrt{\beta/\alpha} - 1)x = 1.61 \times 0.24 = 0.39$ (as in equation [13]). In the first period, the terrorists do not consider that they will attack at rate 0.39 in the second period; however, they do discern that the government will hit with rate at most 0.24 in the first period providing the terrorists plan at most 2.46 terror attacks in the first period—planning more attacks in the first period

Figure 4b
Discounted Repeated Games: Chaos ($\theta^G = 0.8, \theta^T = 0.2$)



would provoke the government to hit back much harder in the first period; thus the terrorists maximize their utility by planning 2.46 first-period attacks.

In the second period, the terrorists again ignore the future and attack with rate 0.39, which is their utility-maximizing attack rate in that period (and to the terrorists the value-maximizing attack rate of the entire game as perceived at that time). However, this attack rate is also the hit-inducing threshold λ^+ (see equation [10] and the discussion from equations [8] to [13]). Consequently, in period 2, *the government sits*.

Since the government sits in period 2, the terrorists do not receive any recruitment benefits in period 3. To the terrorists, period 3 (and all subsequent odd-numbered periods) look just like period 1, while period 4 (and all subsequent even-numbered periods) look just like period 2. The terrorists' myopia when $\theta^T = 0$ enables the government to optimally manage the entire game by repeatedly denying recruitment benefits to the terrorists over time. This explains the two-period cycles in Figure 4a.

Figure 4b reports a startling example that results when $\theta^G = 0.80$ and $\theta^T = 0.20$. In this example, the terrorists and the government select attack and hit rates that result in cycles with different periods over time that are also out of phase with each other. Peak government hit rates are followed by peak terror-attack rates, with peak hit rates higher in the middle periods than at either the beginning or the ending phases of the game.

While the pattern of attacks and hits certainly looks more chaotic over time when θ^G exceeds θ^T , the overall level of violence is actually much higher in the

stationary hit equilibria. In the examples that generate cycles, the terrorists are typically held to attack rates of fewer than 2.5 per period while the government hit rates also remain fewer than 2.5. In contrast, examples that lead to stationary hit equilibria lead to much higher attack and hit rates in the neighborhood of 7 and 4.2, respectively. Thus, if the government values the future more than the terrorists, one can expect low-level (and perhaps chaotic) cycles of violence whereby the attack and hit rates fluctuate over time, whereas if the terrorists value the future more highly, the level of violence is higher but the pattern of violence is more regular.

Discussion

The analyses in this article address sustained (counter-) terror campaigns such as the second intifada. Starting with the two-period model, the knowledge that hits in the first period provide terrorist benefits via recruitment in the second period can deter the government from hitting hard initially. Under such de-escalation scenarios, the government is willing to tolerate moderately high attack rates in the first period to keep potential recruitment and thus the attack rate low in the second period. Alternatively, recruitment combined with somewhat but by no means perfectly effective hitting can lead both terrorists and the government to escalate the level of violence from the first period to the second. In these escalation scenarios, the model suggests something interesting: even when the government treats all civilian casualties equally, and even when the government knows that hitting can lead to downstream terrorism via recruitment, hitting can still be optimal. Prior assessments of the efficacy of targeted killings in the Israeli–Palestinian conflict suggested that hits made matters worse because of this boomerang or recruitment effect (Kaplan et al. 2005; Kaplan, Mintz, and Mishal 2006). To our knowledge, the analysis in the present article is the first that suggests circumstances under which hitting serves to maximize the number of (Palestinian plus Israeli) civilian lives saved.

The multiperiod models capture the significance (and value) of patience in a sustained (counter-) terrorism conflict. Having more patience, or equivalently valuing the future more highly, clearly provides an advantage in our model. When the terrorists are more patient than the government, the stationary hit equilibria provide stable but higher levels of violence. Alternatively, when the government accounts for the future more, even though the resulting pattern of violence exhibits cyclic or chaotic behavior in the terror attack and government hit rates, the overall level of violence is much lower. When both the government and the terrorists value the future equally, the terrorists retain the upper hand because of their overall first-mover advantage in the game. For one, these results show that stability in a sustained conflict (in the sense of constant terror-attack and hitting rates) is not necessarily a good thing. The results also highlight the importance of government's weighing the downstream effects of its current actions more heavily than its

terrorist adversaries. Many would argue that government is less patient because of electoral pressure, while terrorists are more patient, as they are tenured for life.

All of our models are captives of our assumptions, so one must be careful not to overinterpret our results. We have assumed perfect information in our models; combined with the strict sequence of terrorist decision making followed by government play in each period, this assumption amounts to granting the government perfect intelligence (and perfect terrorist recognition that the government has such intelligence). This assumption was also made in the attacker–defender models of infrastructure defense of Brown et al. (2006), where it is argued that information governing the operation of public facilities and utilities is indeed public knowledge. In the Israeli–Palestinian conflict, it has been argued elsewhere that Israeli intelligence regarding terrorist activity emanating from the West Bank is very good, thus perhaps the assumption of perfect information is less objectionable than it might be in other situations. Nonetheless, relaxing the perfect-information assumption by introducing noisy signals into the analysis is one extension that could be studied.

In our analysis, we treated the terrorists as a homogeneous group. In the actual intifada, several organizations (including Hamas, the Islamic Jihad, and the Al Aqsa Martyrs Brigades) were involved and competed with each other in addition to targeting Israeli civilians (Gupta and Mundra 2005).

Our multiperiod model was possible to analyze because of the one-period state-dependence structure: government actions in period t depend only on terrorist decisions in that same period, whereas terrorist decision making in period t depends only on government actions in period $t - 1$. Having the government behave in such a fashion is consistent with the stated policies governing targeted killings in Israel (David 2003; Guiora 2004; Harel and Alon 2002; Katz 2006), though many would argue that past actions have played a role in some assassinations (e.g., the targeted killings of Ahmed Yassin and Abdel Aziz Ratissi, the founder and subsequent leader of Hamas). Similarly, though Israeli hits influenced downstream terrorist decision making in our model, the time lag is only one period; prior research suggests that the reach of targeted killings on downstream terror attacks could be much longer lasting (Kaplan et al 2005; Kaplan, Mintz, and Mishal 2006). Allowing a greater role for history in the dynamic model would complicate the analysis, but perhaps a richer set of model behaviors would emerge.

In spite of these shortcomings, we believe the model results are encouraging and suggestive of the application of game theory to operational decision making in (counter-) terror campaigns. In particular, the evaluation of counterterror measures is an important task facing homeland security and counterterrorism officials. Game theory seems ideally suited for filtering data regarding planned, prevented, and executed terror attacks along with the array of countermeasures deployed. The models of this article perhaps represent a first step in that direction.

References

- Abrahms, Max. 2004. Are terrorists really rational? The Palestinian example. *Orbis* 48 (3): 533-49.
- Abrahms, Max. 2007. Why terrorism does not work. *International Security* 31(2): 42-78.
- Al-Hajjar, Yusef. 2004. " Hamas arch-terrorist Muhammed Deif: An interview." *Jerusalem Post*, March 25, 2004. <http://tinyurl.com/y7u9gp>.
- Arce, Daniel G., and Todd Sandler. 2005. Counterterrorism: A game-theoretic analysis. *Journal of Conflict Resolution* 49 (2): 183-200.
- Atran, Scott. 2003. Genesis of suicide terrorism. *Science* 299 (5612): 1534-39.
- Brophy-Baermann, Bryan, and John A. C. Conybeare. 1994. Retaliating against terrorism: Rational expectations and the optimality of rules versus discretion. *American Journal of Political Science* 38 (1): 196-210.
- Brown, Gerald, Matthew Carlyle, Javier Salmeron, and Kevin Wood. 2006. Defending critical infrastructure. *Interfaces* 36 (6): 530-44.
- Bueno de Mesquita, Ethan. 2005a. The quality of terror. *American Journal of Political Science* 49 (3): 515-30.
- Bueno de Mesquita, Ethan. 2005b. The terrorist endgame: A model with moral hazard and learning. *Journal of Conflict Resolution* 49 (2): 237-58.
- David, Steven R. 2003. Israel's policy of targeted killing. *Ethics and International Affairs* 17 (1): 111-26.
- Dershowitz, Alan M. 2004. Killing terrorist chieftains is legal. *Jerusalem Post*, April 23.
- Enders, Walter, and Todd Sandler. 1993. The effectiveness of antiterrorism policies: A vector-autoregression-intervention analysis. *American Political Science Review* 87 (4): 829-44.
- Enders, Walter, Todd Sandler, and Jon Cauley. 1990. UN conventions, technology and retaliation in the fight against terrorism: An economic evaluation. *Terrorism and Political Violence* 2 (1): 83-105.
- Ganor, Boaz. 2005. *The counter-terrorism puzzle: a guide for decision makers*. Herzliya, Israel: IDC.
- Gross, Michael L. 2003. Fighting by other means in the Mideast: A critical analysis of Israel's assassination policy. *Political Studies* 51 (2): 350-68.
- Guiora, Amos N. 2004. Targeted killing as active self-defense. *Case Western Reserve Journal of International Law* 36 (2/3): 319-34.
- Gupta, Dipak K., and Kusum Mundra. 2005. Suicide bombing as a strategic weapon: An empirical investigation of Hamas and Islamic Jihad. *Terrorism and Political Violence* 17 (4): 573-98.
- Harel, Amos, and Gideon Alon. 2002. IDF lawyers set "conditions" for assassination policy. *Haaretz* (English edition), February 2, 2002. <http://tinyurl.com/y8xusl>.
- Heal, Geoffrey, and Howard Kunreuther. 2005. IDS models of airline security. *Journal of Conflict Resolution* 49 (2): 201-17.
- Izenberg, Dan. 2006. High Court rules targeted killings are legal if they meet proper criteria. *Jerusalem Post*, Dec. 15.
- Kaplan, Edward H., Alex Mintz, and Shaul Mishal. 2006. Tactical prevention of suicide bombings in Israel. *Interfaces* 36 (6): 553-61.
- Kaplan, Edward H., Alex Mintz, Shaul Mishal, and Claudio Samban. 2005. What happened to suicide bombings in Israel? Insights from a terror stock model. *Studies in Conflict and Terrorism* 28 (3): 225-35.
- Katz, Yaakov. 2006. Defense: Pinpointing the problem. *Jerusalem Post*, June 23.
- Maskin, Eric, and Jean Tirole. 1987. A theory of dynamic oligopoly, III: Cournot competition. *European Economic Review* 31 (4): 947-68.
- Maskin, Eric, and Jean Tirole. 1988. A theory of dynamic oligopoly, I: Overview and quantity competition with large fixed costs. *Econometrica* 56 (3): 549-69.
- Pape, Robert A. 2003. The strategic logic of suicide terrorism. *American Political Science Review* 97 (3): 1-19.

- Rosendorff, Peter, and Todd Sandler. 2004. Too much of a good thing? The proactive response dilemma. *Journal of Conflict Resolution* 48 (5): 657-71.
- Sandler, Todd, and Daniel G. Arce. 2007. Terrorism: A game-theoretic approach. In *Handbook of defense economics*. Vol. 2, *Defense in a Globalized World*, ed. Todd Sandler and Keith Hartley, 775-814. Amsterdam: North Holland.
- Siqueira, Kevin, and Todd Sandler. 2006. Terrorists versus the government: Strategic interaction, support and sponsorship. *Journal of Conflict Resolution* 50 (6): 878-98.
- Wein, Lawrence M., and Manas Baveja. 2005. Using fingerprint image quality to improve the identification performance of the U.S. Visitor and Immigrant Status Indicator Technology Program. *Proceedings of the National Academy of Sciences of the United States of America* 102 (21): 7772-75.

Erratum

On page 786 of Daniel Jacobson and Edward H. Kaplan's article, "Suicide Bombings and Targeted Killings in (Counter-) Terror Games," which appeared in the October 2007 issue of the *Journal of Conflict Resolution*, the mathematical expression in the following sentence inaccurately appeared as:

"In each period, the government sits while the terrorists attack at the rate that maximizes their per-period payoff (given by $\min(\lambda_2(0), \lambda^+)$ from equations [9,10]."

The expression should have appeared as:

$$\min(\lambda'_2(0), \lambda^+)$$